For constant phase by all paths, then we must have the equation

$$\beta_0 f + \beta_1 t = \beta_0[(f + z)^2 + r^2]^{1/2} + \beta_1(t - z)$$

or

$$f = [(f + z)^2 + r^2]^{1/2} - (\mu_r\epsilon_r)^{1/2} z$$

or

$$r^2 = (\mu_r\epsilon_r - 1)z^2 + 2f[(\mu_r\epsilon_r)^{1/2} - 1]z$$

This equation may be recognized as the equation of a hyperboloid of revolution (in rectangular coordinates $r^2 = x^2 + y^2$). These results show that a lens with a plane surface and a hyperbolic surface may be designed to convert spherical wavefronts to plane wavefronts in the aperture.

Lenses have a few special problems. In general, there will be a reflection from both interfaces of a lens. Let us consider the reflection from the plane surface first. If the system is employed as a transmitter, the energy reflected from the plane face will pass back through the lens surface and be focused at the feed. Thus, in addition to the loss in efficiency, the feed system will be mismatched. Some type of nonreflecting surface coating is clearly desirable. A quarter-wave thickness having the geometric mean intrinsic impedance will eliminate this reflection. Corrugations or other loadings may also be employed as matching layers.

The effect of the reflection at the curved surface is more complicated. The amount of the reflection depends on the angle of incidence to the surface and the polarization with respect to the plane of incidence (good approximate results can be obtained with the usual Fresnel formulas). The net result is that amplitude distribution on the plane face may depend significantly on these reflections from the back face; for example, the effect of the Brewster angle may be seen. Consequently, it is usually desirable to eliminate the reflections from the curved lens surface as well as the plane one. A quarter-wave matching section added to the curved surface will work satisfactorily.

The inherent loss and structural instability of some dielectric lenses can be eliminated by a different approach to lens construction. In particular, the lens may be constructed from metal instead of dielectric. With this approach, the required phase delays have been obtained from "waveguide lenses" as well as with artificial dielectrics made of metal disks, rods, and spheres. See, for example, the chapter in Jasik[1] by Seymour Cohn.

[1] H. Jasik, "Antenna Engineering Handbook," chap. 14, pp. 14-21 to 14-41, McGraw-Hill Book Company, New York, 1961.

Antenna Engineering, Walter L. Weeks
McGraw-Hill

7. ANTENNAS FOR MULTIPLE FREQUENCIES

In practice, there are usually rather stringent space and cost limitations on antenna systems; moreover, for flexibility, it is usually necessary to provide for operation at several frequencies. This implies that antennas must be designed so that they will operate, often with almost identical performance, at several frequencies. However, as we have seen throughout this book, antenna performance, with regard to both input impedance and radiation pattern, is characterized by the antenna dimensions measured in wavelengths; the performance for a simple structure is then inherently frequency-dependent. In the applications, there are two broad classes of requirements: sometimes, an antenna must operate only in two or, at most, a relatively small number of discrete narrow bands; on the other hand, some antennas must provide an almost continuous frequency coverage over a frequency range that may cover from 2 to 10 octaves. For the former applications, it is appropriate to employ what might be called "spot-band" antennas. For the latter, a more elaborate "frequency-independent" antenna is required.

7.1 SPOT-BAND ANTENNAS

Spot-band antennas are designed to operate only within a small number of narrow-frequency bands. The performance at other frequencies is usually erratic and widely variable but is of no concern in the specific application. The guiding principle in the design is usually the incorporation of structural features that serve to enhance or suppress the excitation of different frequencies. Perhaps the simplest example of this technique is the design of a simple dipole for operation at two frequencies. The idea is to incorporate a high impedance to suppress currents on a part of the structure. Thus, for example, a dipole may be cut with a length

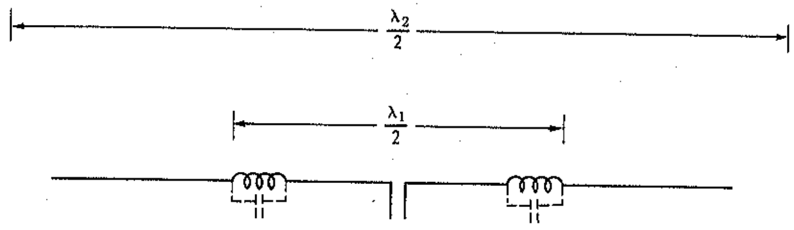

Fig. 7.1 Idea of simple two-band antenna. Impedance of L is very large (choke) at frequency f_1, but much less at frequency f_2 (providing only a loading effect).

slightly less than a half wavelength at the higher frequency. This is separated from an additional length of wire or rod by a choke (see Fig. 7.1). The choke inductance is selected so that the impedance is very high at the high frequency, but low at the low frequency. Alternatively, a capacitor may be added so that the high impedance is obtained by a parallel-resonant circuit. Thus, at the low frequency, the structure also closely approximates a half-wave dipole, differing only in the small inductive loading. As a result, the performance can be nearly the same at two well-separated frequency bands; of course, the scheme will not operate satisfactorily at two arbitrarily related frequencies.

Another approach is to include two or more radiators but to employ transmission-line sections so as to diplex the signals into the appropriate radiator. For example, suppose the application is suited to a two-element Yagi antenna but calls for operation at two frequency bands. In this case, the main transmission line might be branched into lines leading to the two radiating elements (Fig. 7.2). Then, transmission-line tuning stubs are introduced into each of the branched lines, so as to provide, at the branch point, an open circuit at one frequency and a match at the other frequency (looking into that particular line). This provides for the excitation of one or the other of the dipoles. Also, a short circuit at the feed point of that dipole attached to the unexcited branch may be provided by selecting the length of line to the tuning stubs or with another set of stubs. The net result is a structure which at one frequency is a driven dipole with a parasitic director and at the other frequency a driven dipole with a parasitic reflector. This technique is appropriate as long as the two frequency bands are not too widely separated.

Exercise Work out the details of the design for the two-band antenna suggested in Fig. 7.2. Assume a 50-ohm line and take the frequencies of operation to be 60 MHz and 80 MHz.

A somewhat more complicated antenna design may be illustrated with the description of a dual-channel Yagi-type antenna for a pair of tele-

vision channels. Consider for example a Yagi antenna having four directors and a reflector, as shown in Fig. 7.3. The directors are cut so as to give best operation at the higher frequency band, and the reflector is cut to give best operation at the lower frequency band. The driven element is a folded dipole, basically cut for operation at the higher frequency. However, on the unfed side of the folded dipole, a tuning-stub arrangement is connected which somewhat resembles a large hairpin. The length of the open-circuited stub is adjusted to be a quarter wavelength at the upper frequency so that the open transfers into a short circuit at the junction. The curved (short-circuited) side of the hairpin provides sufficient length so that, with allowances for the capacitive effect of the open stub, the folded dipole is also resonant at the lower frequency.

As an example of another type of broadbanding, consider an antenna type described by J. T. Bolljahn in U.S. Patent No. 2,505,751. This structure was developed as an aircraft antenna, aimed to cover almost an octave in the lower part of the UHF band. The antenna consists of a "partial sleeve" monopole, made up of a driven element and two short

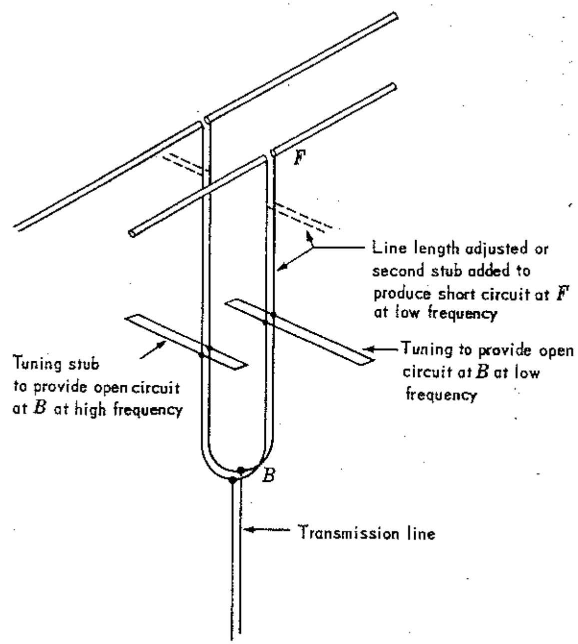

Fig. 7.2 A two-band Yagi array with tuning-stub diplexer.

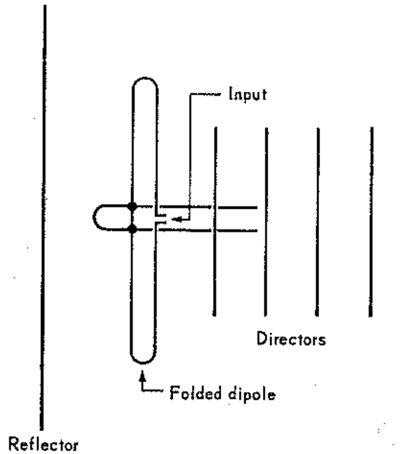

Fig. 7.3 Two-channel Yagi array, with hairpin-tuning stubs on folded dipole.

parasites, as shown in Fig. 7.4. With the particular dimensions shown, the structure provides a VSWR of less than 1.8 on a 50-ohm line over the frequency band from 310 to 510 MHz.

The other main technique for designing spot-band antennas is to attempt to fit antenna systems for the different frequencies into the same

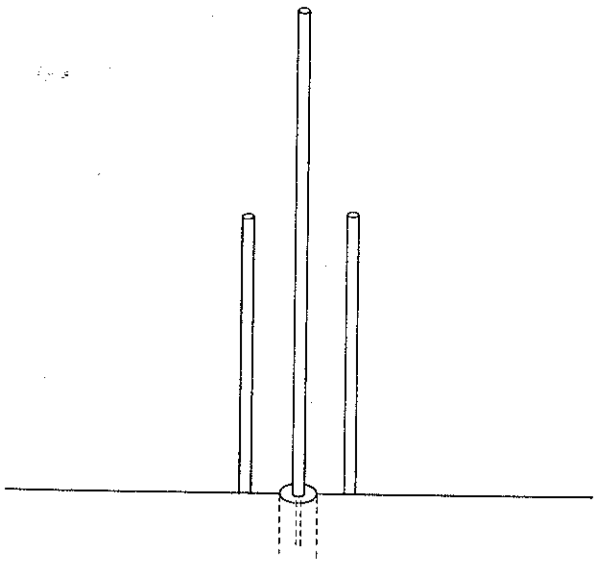

Fig. 7.4 Open-sleeve antenna for operation between 310 and 520 MHz. Longer element is $8\frac{11}{16}$ in.; shorter ones are $5\frac{1}{8}$ in.

space. This process of interleaving and interlacing usually requires a large measure of ingenuity along with compromises in order to prevent one antenna system from drastically influencing the performance of another.

A large step forward in antenna engineering came with the development of "frequency-independent" antennas, the subject of the next section. This section includes somewhat more than the usual amount of informal history; this detail is presented as a case history of a not-so-straightforward antenna development. It is also presented in the belief that this information aids in the understanding of the great variety of antennas of this type now employed.

7.2 FREQUENCY-INDEPENDENT ANTENNAS[1]

The research work which led to the development of antennas whose performance is almost independent of frequency was carried out mainly at the University of Illinois in the period from 1955 to 1958. The work, along with several other projects, was sponsored by the Air Force in order to relieve the problems associated with the increasing numbers of different electromagnetic systems and equipment being carried on high-speed military aircraft. So many different antennas were required that the finding of locations for the antennas was a very serious problem. It was recognized that the problem would be relieved if a given antenna could serve several systems and frequencies, and consequently the Air Force sponsored a research program on the general subject of broadband antennas.

Out of this research work came many different novel, unconventional structures, few of which are such as to permit a tractable mathematical description. It seems that the best way to provide some insight into the operation and design of these novel broadband structures is to trace their historical development. This is done in the next paragraphs.

In connection with the sponsored research work on broadband antennas, Professor V. H. Rumsey, then antenna laboratory director at the University of Illinois, asked himself the question: "What is it that makes an antenna sensitive to frequency?" In thinking about this, he noticed that the features which introduce the frequency dependence are the *characteristic lengths* of the structure. Antenna performance is generally a function of length/wavelength. On the other hand, by the principle of modeling or scaling, to ensure that a given type of structure has the same performance at different frequencies, it is only necessary to scale the size of the structure in the ratio of the frequencies. Thus, Rumsey concluded that the structural feature required for frequency-independent operation is the absence of characteristic lengths. With this feature, a structure could be self-scaling.

[1] E. C. Jordan, G. A. Deschamps, J. D. Dyson, and P. E. Mayes, *IEEE Spectrum*, **1**:58 (April, 1964).

But what kind of physical structure is there that has no physical lengths? Rumsey's answer was that the structure should be completely described by angles. Thus, he put forward the "angle concept," which said, essentially, that a structure whose shape is defined by angles alone, with no characteristic lengths, should be a frequency-independent structure. In looking for structures that can be defined by angles alone, the first that come to mind are the infinite biconical antenna (or transmission-line) and the infinite bifin (bow-tie) antenna. However, practical versions of these structures are obviously finite in size, and although these structures do have comparatively broadband tendencies, the truncation to a finite size introduces a characteristic length, and this destroys the frequency-independent behavior. In thinking about other structures, Rumsey recalled the logarithmic or equiangular spiral, whose curve in a plane is given by the equation $\rho = \rho_0 e^{a\phi}$; or, if the curve is developed on a cone of angle $\theta = \theta_0$, its equation is $r = r_0 e^{a\phi}$. He noted that, except for a rotation in space about the axis of the spiral, this structure, if infinite, should look the same at any frequency. Although the infinite structure could not be built, Rumsey reasoned that if the spiral were excited at the origin, the currents on the arms might fall off rapidly enough that, at least through a wide band of frequencies, the fact that the structure must be finite in size would not matter. Moreover, earlier Air Force experiments reported by E. Turner had indicated that an archimedean spiral had promise as a broadband antenna. Rumsey therefore asked J. Dyson, then a graduate student employed by the laboratory, to build and test an equiangular spiral antenna.

At the same time, Rumsey recalled another fact that he had seen earlier. Namely, he recalled from Booker's work the relation between the impedance of a slot antenna and the complementary dipole antenna (page 258), $Z_{slot} \cdot Z_{dipole} = (1/4)(\mu/\epsilon)$. This equation implies that if the slot and the complementary dipole could be made to look the same, then Z_{slot} should equal Z_{dipole}, and therefore the input impedance of either should be a constant, independent of frequency. If the slot and the complementary dipole are the same, it is appropriate to call the structures *self-complementary*. Examples of self-complementary structures are shown in Fig. 7.5. Self-complementary structures of one class may be constructed by drawing an arbitrary (nonoverlapping) curve from the origin and letting it extend to infinity. This curve is then rotated 90° about the origin and redrawn; it is rotated another 90° and redrawn, and so on until it is back to its original position. The plane is thereby split into four parts. If the alternate parts are filled in with metal, the resulting structure is self-complementary.

Rumsey noticed that the equiangular spiral could be made from sheet metal in such a way that it would be self-complementary, and he asked

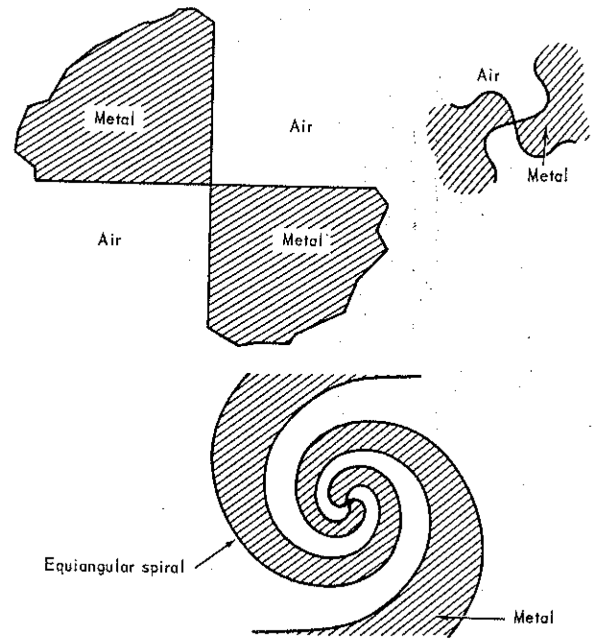

Fig. 7.5 Self-complementary structures.

Dyson to build this feature into the first model for testing. The results were striking. The first models had bandwidths of several octaves in the sense that they had almost constant input impedance and a nearly constant radiation pattern (constant except for a rotation about the axis of the spiral). Moreover, the patterns were observed to be circularly polarized. The current distribution along the arms of the spiral was measured, and it was found that (1) the currents fall off faster than $1/r$, and (2) at all frequencies within the band over which the performance was observed to be constant, the current amplitude was substantially zero at or before the point of truncation. At frequencies such that the diameter of the truncated spiral is approximately equal to a wavelength, the currents at the point of truncation begin to be significant and the performance begins to deteriorate. On the other hand, the upper limit on the frequency-independent operation is determined by the accuracy with which the feed region is (or can be) constructed.

When the spiral is operating within its frequency-independent band, it is self-scaling. Thus, its input impedance should be independent of frequency, even without the self-complementary feature discussed above.

Consequently, this feature was eliminated on later experimental models, and it was found that the bandwidth was indeed maintained, as expected (although the VSWR is more nearly constant with the self-complementary structures).

The spiral antenna is typically excited by running a small coaxial line from the outer extremities along one of the arms of the spiral, into the origin. There the center conductor is led out and joined to the other arm of the spiral. With this type of feed, the structure provides its own balun. Usually, a dummy cable is soldered on the second arm in order to preserve the symmetry. Further details on the design of spiral antennas can be found in the publications of Dyson.[1]

The work on spirals was a significant advance in antenna art, but it certainly did not solve all broadband-antenna problems. The radiation patterns of the plane spirals are broad and bidirectional with the maxima along the axis perpendicular to the sheet metal, and are circularly polarized. Consequently, R. H. DuHamel (then a research assistant professor employed by the University of Illinois Antenna Laboratory) addressed himself to the problem of designing a broadband antenna with linear polarization. He realized that the bifin or bow-tie antenna could be constructed in a self-complementary fashion and of course that it radiates linear polarization. But he also realized that the bandwidth of the bifin was limited because of the truncation, or more particularly, because the currents were not negligible at the point of truncation. Consequently, the problem as DuHamel saw it was to somehow alter the bow-tie structure in such a way as to cause the currents to fall off with distance from the feed point more rapidly than usual. His method for accomplishing this was to introduce discontinuities, for example, teeth, into the fins in an attempt to increase the radiation and speed up the decay of current. But the question he had to ask himself was, "How should the teeth be designed?" DuHamel decided that he should adhere to Rumsey's angle concept as far as possible. Consequently, he decided to cut the teeth along circular arcs and let the length of the arcs be determined by an angle (see Fig. 7.6). However, this did not fix the tooth spacing, since the latter could not be specified by angles alone.

In trying to decide what spacing to use on the teeth, DuHamel noticed that on the equiangular spiral (a successful structure), along a line drawn from the center outward, the spacings from one conductor to the next were in a constant ratio (since the defining curve was $r = r_0 e^{a\phi}$). He therefore considered spacing the teeth in the bifin such that the spacings were in a constant ratio. He accomplished this by choosing the radii of

[1] J. D. Dyson, *IRE Trans.*, **AP-7**:181–187 (April, 1959). *IRE Trans.*, **AP-7**:329–334 (October, 1959). *IEEE Trans.*, **AP-13**:488–499 (July, 1965).

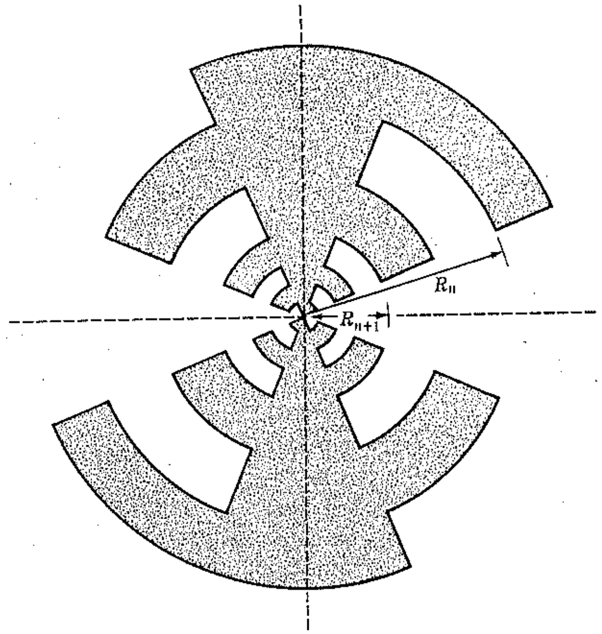

Fig. 7.6 Log-periodic toothed structure (self-complementary).

the circular arcs forming the corresponding parts of the successive teeth such that they were in a constant ratio, $R_{n+1}/R_n = \tau$. He recognized that the structure would not necessarily be frequency-independent but that, on the other hand, the performance on an infinite structure would be identical at a discrete number of frequencies. (For example, consider a frequency at which the pth tooth is, say, one-tenth of a wavelength in size. Then, at a lower frequency, the $(p - 1)$st tooth will be one-tenth of a wavelength in size, and all the rest of the structure will be scaled accordingly; therefore the performance at the lower frequency will be the same as that at the first frequency.) In fact, if the structure has a performance (E_1, Z_1) at frequency f_1, the performance should be identical at frequencies τf_1, $\tau^2 f_1$, $\tau^3 f_1$, and so on as long as the structure is modeled accurately at the feed point and is effectively infinite in size (i.e., current zero at the point of truncation). Again, τ is the common ratio of distances. The frequencies at which the performance should be identical are related by the equation $f_n = f_{n+1}\tau$, or $\log f_{n+1} = \log f_n + \log (1/\tau)$. Inspection of this latter equation shows that the performance is a periodic function of the logarithm of the frequency (i.e., the frequencies at which the performance is the same are spaced equally when plotted on log paper). Thus, these types of structures were subsequently named *log-periodic* antennas.

Although initially it was by no means clear what the performance would be as a continuous function of frequency, DuHamel decided that, since the percentage bandwidth would also be independent of frequency, it should be well worth the effort to investigate the structures experimentally. Thus, toothed structures of the type indicated in Fig. 7.6 were built and tested. Initially, the self-complementary feature was built into all models, to guarantee a constant input impedance on the infinite structure. The experiments showed that on structures with large enough teeth, the current decay was rapid and that the log-periodic frequency behavior was verified. But perhaps more important, on some of the structures, it was found that the change in the radiation pattern with frequency at frequencies between the log-periodic frequencies was relatively minor. That is, the structures were self-scaling with only minor variations between periods. Eventually it was learned that the self-complementary feature is not essential to acceptable impedance bandwidth.

Many structures of this type were built and tested. Some were less successful (i.e., frequency-independent) than others, and it was noticed that some of the more successful had a polarization whose strongest linear component was perpendicular to that normally radiated by a bow-tie

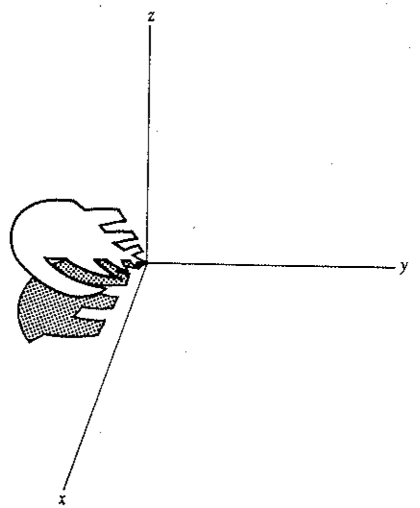

Fig. 7.7 Nonplanar log-periodic antenna.

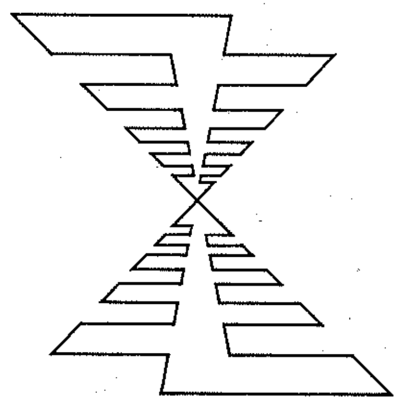

Fig. 7.8 Trapezoidal toothed log-periodic structure.

antenna. This indicated that significant currents were flowing along the teeth.

One of the objectives, namely a broadband linearly polarized antenna, was attained with the invention of the toothed log-periodic structures. However, the patterns were still broad and bidirectional, normal to the plane of the sheet metal from which the antennas were constructed. The next step forward came when D. E. Isbell (then a laboratory assistant working with DuHamel), in an attempt to produce a unidirectional pattern, decided to ignore the planar restrictions suggested by Babinet's principle. He modified one of the successful planar structures by folding it into a wedge. To the surprise of almost everyone in the laboratory except Isbell, the experiments with the wedge structure (Fig. 7.7) showed that it radiated unidirectionally, with the maximum in the radiation pattern off the tip (feed point) of the structure. Moreover, the bandwidth was found to be practically the same as that of the planar structure (although the impedance levels were lower). When oriented as in Fig. 7.7, the polarization was predominantly in the x direction. It was found subsequently that the teeth could be cut straight as well as curved (Fig. 7.8) to give what was called a trapezoidal toothed structure.

About this time it began to be evident that most of the current excitation and radiation were associated with teeth whose lengths were in the vicinity of a quarter wavelength. Still later, DuHamel recognized that the sheet-metal structures could be simulated with wires or tubes which outline the periphery of the sheet structures. The tube type of construction was later developed by DuHamel and coworkers at Collins Radio Company into a class of commercial antennas for application in the HF band, the type Collins 237A-1 (Fig. 7.9). Antennas of this type, made of both sheet metal and wires, were also employed as feeds for

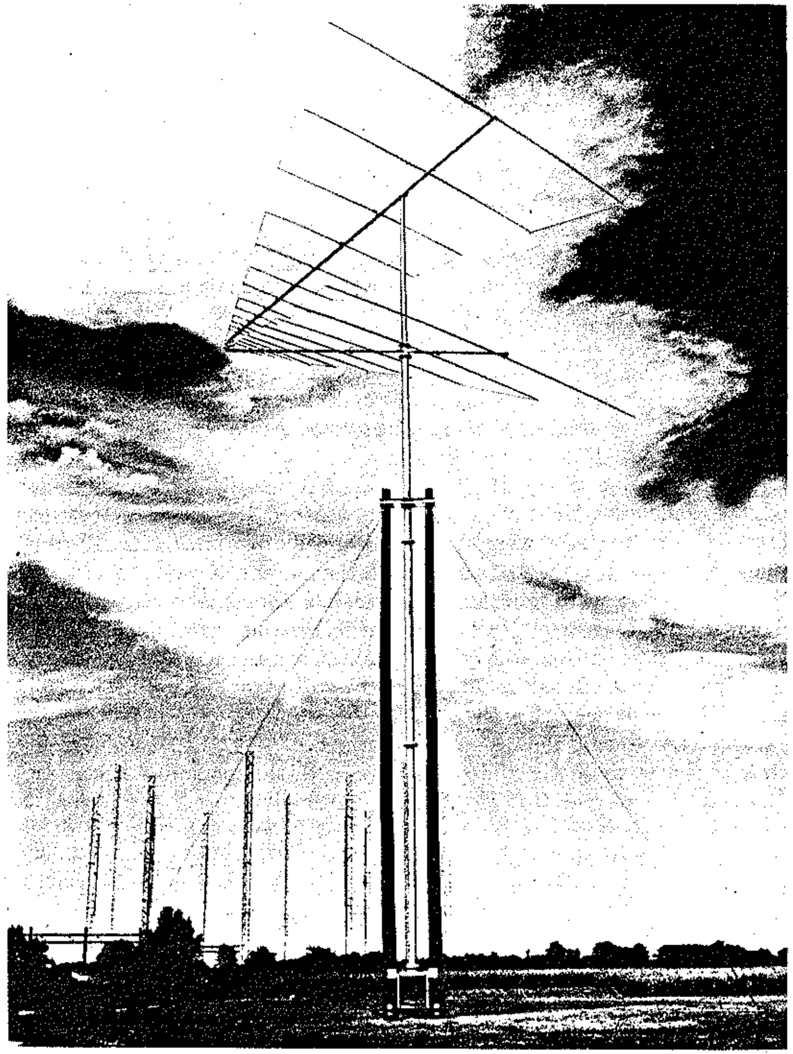

Fig. 7.9 A rotatable nonplanar log-periodic antenna system for 6.5 to 60 MHz. (Collins Radio Company 237A-1.)

parabolic reflectors. Typical gain for log-periodic antennas of the type in Figs. 7.8 and 7.9 is about 8 to 10 db, with a front-to-back ratio of 15 and an H plane beamwidth of 80 to 110°. Further details are available in the literature.[1]

The next major step came with Isbell's invention of the log periodic dipole array. His work was motivated by the desire to develop broadband arrays of more conventional construction. Thus he decided to build and test an antenna array constructed of conventional wirelike elements; however, the lengths of the elements were determined by an angle α as before, and the spacings were such as to give the log-periodic type of behavior; that is, successive distances between the apex and the elements were in a constant ratio, $R_{n+1}/R_n = \tau$. With this general structural type in mind, the big question was how to excite the elements in this array. Taking a hard look at a successful trapezoidal tooth antenna (Fig. 7.8), Isbell saw that when this structure was bent over into a wedge to give the unidirectional pattern, the center portion on each arm could be regarded as a transmission line feeding the teeth as radiators; the radiating teeth were simply connected in shunt on this line. For the new array, Isbell decided to ignore the fact that the feed line should, strictly speaking, be conical and expanding in radius and to try instead an ordinary parallel-wire line as the feeder transmission line for the teeth. Taking another hard look at the trapezoidal toothed structure, he noted that if the two arms were folded back so as to be almost parallel, then pairs of equal-length elements would almost be lined up as dipoles. However, he noticed also that on a given arm of the structure, successive teeth came out from the centerline in opposite directions. Consequently, he decided that in order to simulate the toothed structure, the dipoles should be connected to the parallel-wire line in such a way that the successive elements came out from the line in opposite directions. Isbell accomplished this with the type of construction indicated in Fig. 7.10*a*, in which the two halves of the dipoles are offset slightly. Note also that the parallel-wire line was itself excited by means of a coaxial line which was led through one of the rods making up the parallel-wire line. The center conductor of the coax is led over to the other member of the parallel-wire pair. This method of feeding provides a built-in broadband balun. Isbell also recognized immediately that his method of connecting the dipoles to the line is equivalent to crisscrossing the wire line between the elements as indicated in Fig. 7.10*b*, which clearly introduces a 180° phase shift between elements. The experiments with the structure demon-

[1] R. H. DuHamel and D. E. Isbell, *IRE Natl. Conv. Record*, **pt. 1**:119–128 (1957). R. H. DuHamel and F. R. Ore, *IRE Natl. Conv. Record*, **pt. 1**:139–151 (1958). R. H. DuHamel and D. G. Berry, *IRE WESCON Conv. Record*, **pt. 1**:161–174 (1958). *Ibid.*, *IRE Natl. Conv. Record*, **pt. 1**:42 (1959).

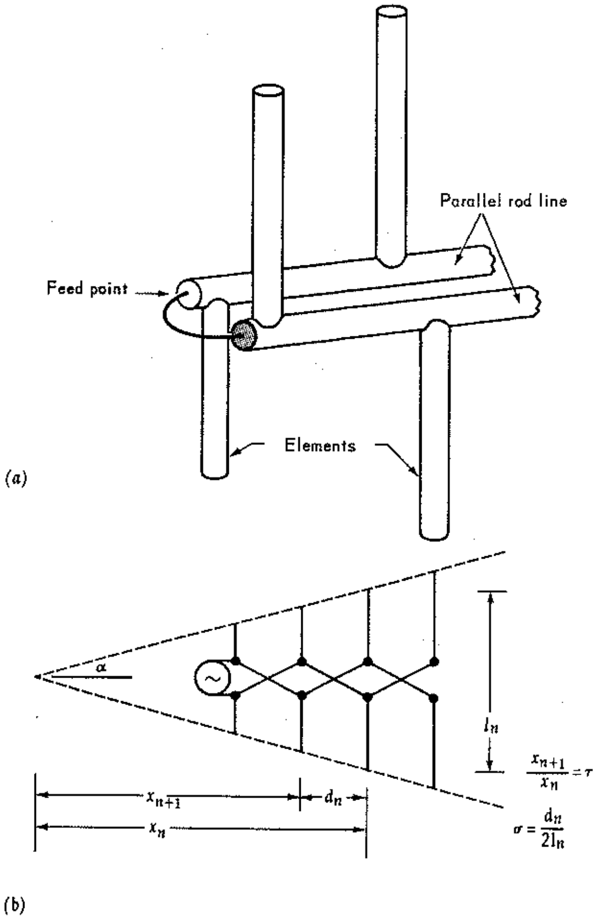

Fig. 7.10 Log-periodic dipole construction.

strated that in a certain range of values for τ and α, the structure was indeed a broadband log-periodic structure with a unidirectional pattern. Isbell also demonstrated experimentally that most of the radiation was coming from those dipole elements which were in the vicinity of a half wavelength long and that the currents and voltages at the large end of the structure were negligible within the operating band of frequencies. Finally, it was shown once again that the operating band of frequencies was bounded on the high side by frequencies corresponding to the size

of the smallest elements (and the feed-line size) and on the low side by the frequencies at which the largest dipole element is about a half wavelength long.[1]

A careful and extremely valuable analysis of the log-periodic dipole array was made by R. L. Carrel in a doctoral dissertation. The physical makeup of the log-periodic array, unlike its predecessors, is such that an analysis of it may be based on more or less conventional theory of linear antennas and transmission lines. The main difficulty is the inherent complication. Carrel's analysis consisted of breaking the overall problem into parts, each of which was programmed for the digital computer. First, making the assumption that the element currents were sinusoidally distributed, he computed in the conventional way the mutual impedances between the dipole elements and the self-impedance of each element. In the second part of the problem, Carrel focused his attention on the parallel-wire transmission line, fed at one end and shunt-loaded with impedances corresponding to the dipole antenna elements having sizes and spacings characteristic of log-periodic dipole arrays; of course, the impedance values came from the first part of his computer program. He carried out (on the digital computer) a circuit type of analysis to find the input impedance, voltages, and currents on the loaded transmission line, together with the base (i.e., input) currents at each antenna element. As the last part of the problem, with the specific values for the magnitude and phase of the currents in the antenna elements, he calculated the radiation patterns. Having developed a systematic computer program, Carrel completed calculations on more than 100 different log-periodic dipole designs. He then compared the results of several of these with corresponding experimental models. The measurements included not only impedances and radiation patterns but also the voltage and current distributions in the structure. The agreement between the computer output and the experimental results was excellent. Carrel's work includes two important features: (1) he presents enough detail concerning the voltage and current distribution on the structures to provide insight into the operation of the antenna; (2) he presents a set of design curves which show how to adjust the dimensions of a structure in order to meet specified design objectives.

First, to better understand the operation of the antenna, let us examine the typical results for voltages and currents on a log-periodic dipole array. Figure 7.11, from Carrel's work,[2] shows the amplitude and phase of the transmission-line voltage (voltage at the base of the elements) as a function of distance along the line (from the apex). In this particular

[1] D. E. Isbell, *IRE Trans.*, **AP-8**:260–267 (May, 1960).
[2] R. L. Carrel, University of Illinois Antenna Lab. Tech Rept. 52, "Analysis and Design of the Log-periodic Dipole Antenna," Contract AF 33(616)-6079.

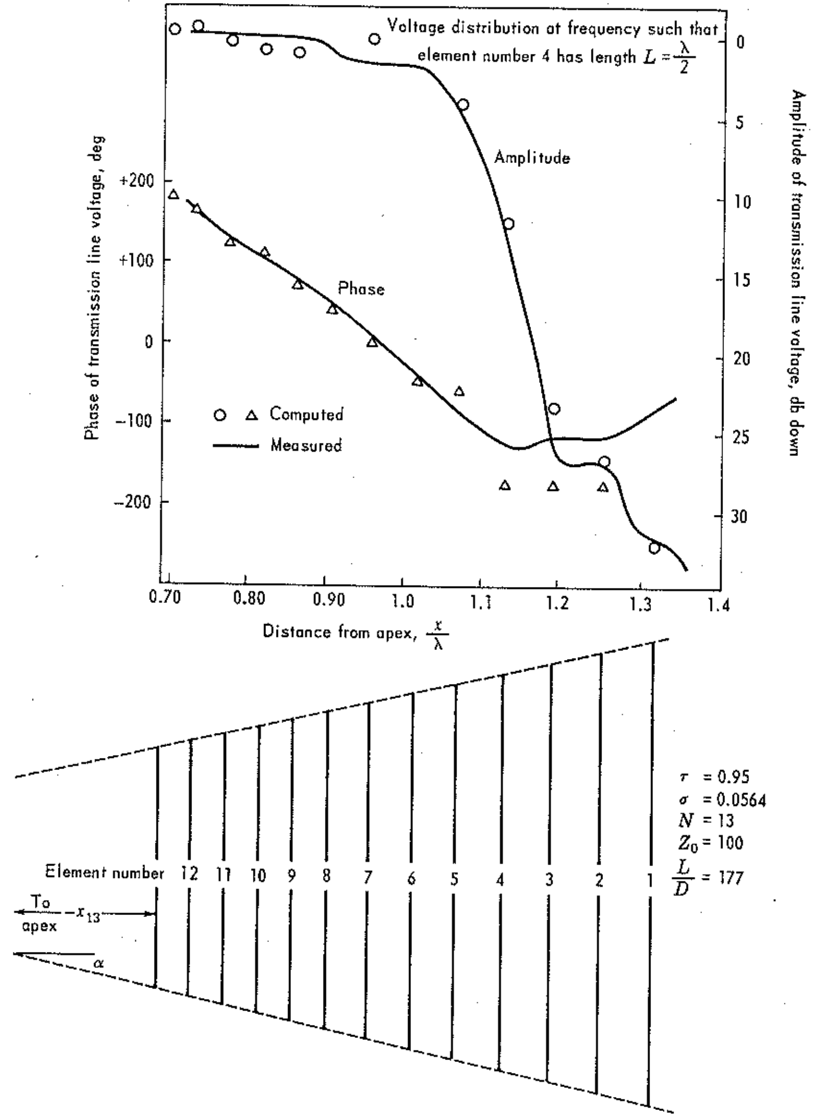

Fig. 7.11 Feeder-line voltage on log-periodic dipole array. Lower figure shows relative position of the elements.

figure, the frequency is such that the element numbered 4 is a half wavelength long. Note in the plot that the voltage is almost constant in amplitude and uniform in phase progression from the point of feed down to about the element numbered 6 (the largest element is numbered 1). This region of constant voltage is called the transmission region at this frequency, since the voltage distribution is almost like that on a matched transmission line. Note, however, that the phase changes by about 150° along a length of line whose length is about a quarter of a free-space wavelength. This means that the phase velocity on the line is only about 0.6 of that of plane waves in space. The decrease in phase velocity is caused by the shunt-capacitive loading of the line by the smaller antenna elements. This loading turns out to be almost constant per unit length, because the larger elements are more widely spaced. Immediately after the element numbered 6, the amplitude of the voltage falls off rapidly; the linear progressive phase variation is also disturbed. The voltage falloff is associated with a relatively strong current excitation in the antenna elements, as will be discussed immediately below. Note finally that the voltage at the largest element is very small, being lower by some 30 db than the voltage at the smaller elements.

Next (Fig. 7.12), consider the currents in the antenna elements. Note that the amplitudes of the currents in elements 7, 6, 5, and 4 are some 5 to 10 db greater than the current amplitudes in the other elements; in fact, the current amplitude in the largest element is too small to plot. The region of relatively high-current amplitudes is termed the "active" region. The fields at the large end of the structure are so small that the fact that the structure does not extend indefinitely is of no consequence at this frequency. The smaller elements are too small to be excited effectively. Next note that the relative phase of the element currents, particularly in the active region, is such that there seems to be a linear progressive phase shift in the direction opposite to that which would occur on an unloaded line. The phasing of the currents in the active elements is in fact suggestive of a wave traveling back toward the feed point. This phasing also accounts for the "backfire" characteristic of the radiation pattern.

As the frequency of operation is changed, the general pattern of voltage and current distribution remains the same, but the active region moves to different elements. In particular, as the frequency is increased, the active region moves in the direction of the shorter elements. When, as a result of the frequency increase, the active region has moved up to the smallest element, the performance begins to change with frequency. Roughly speaking, the high-frequency limit is the frequency at which the shortest element is about a half wavelength long, provided the feed-line separation is small compared to a wavelength. As the frequency is

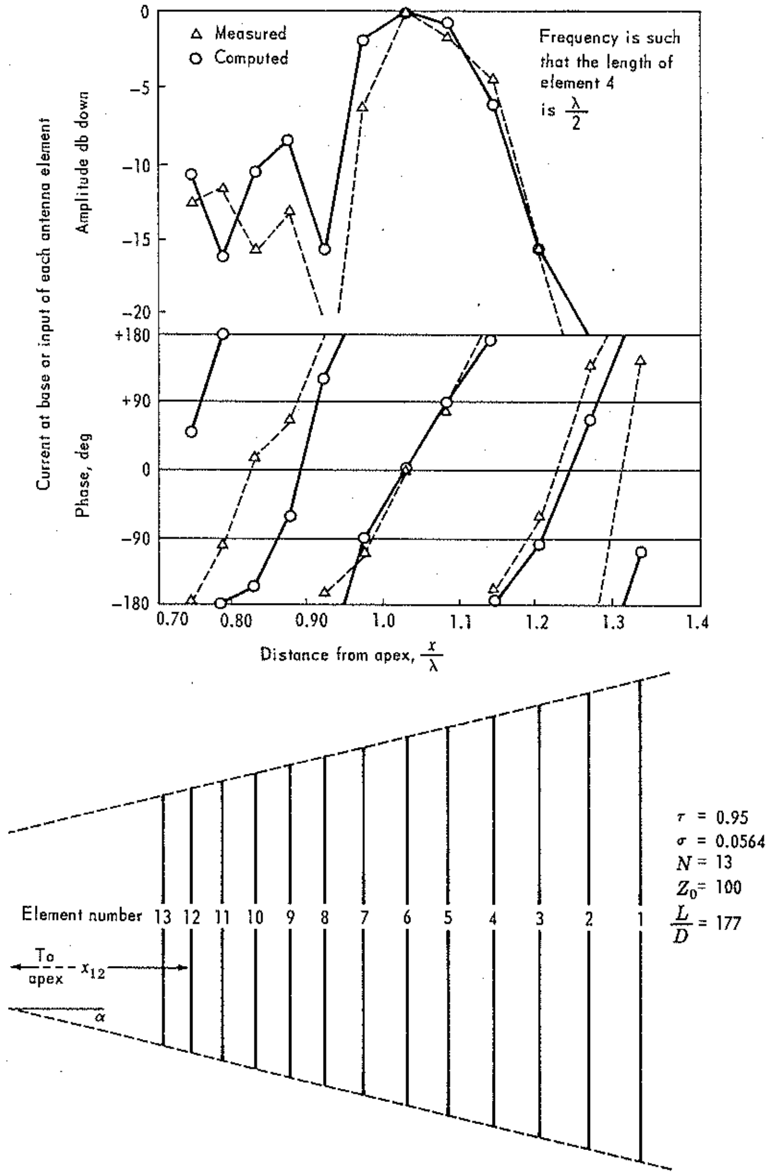

Fig. 7.12 Element input currents on log-periodic dipole array. Lower figure shows relative position of the elements.

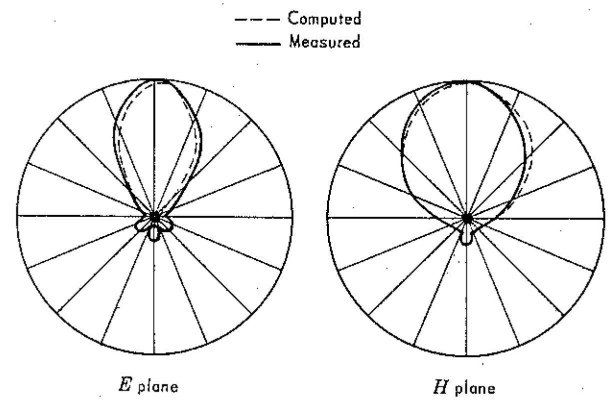

Fig. 7.13 Log-periodic dipole radiation patterns.

decreased, the active region moves toward the longer elements. The performance is satisfactorily constant until the active region reaches the largest element. Roughly speaking, the low-frequency limit is the frequency at which the largest element is a half wavelength long.

Given the element currents and positions, it is a relatively easy matter to compute the radiation pattern. Figure 7.13 shows the pattern of a typical log-periodic antenna which has parameters similar to those for which the voltage and current data are presented. As mentioned above, the direction of the maximum in the radiation pattern is off the small end of the structure, which is opposite to the direction in which the wave starts down the parallel-wire feeder.

As the frequency is varied within the operating bandwidth of the structure, the input impedance values at the feed point, when plotted on a Smith chart, lie on and within a small circle centered on the real axis of the chart. The mean input-resistance level for the structure is designated R_0. This mean resistance level is essentially the characteristic impedance of the equivalent line comprising the transmission-line region. It is found that the actual value, although it is a function of several variables, is only a slowly varying function of the parameter τ. The mean resistance level as a function of σ is shown in Fig. 7.14. Next, Fig. 7.15 shows a typical variation of R_0 with Z_0, the characteristic impedance of the unloaded parallel-rod feeder line. The input-resistance level also depends on the length-to-diameter ratio of the dipole elements, as indicated in Fig. 7.16. In general, it is found that if Z_0 is less than about 75 ohms, then an appreciable amount of power remains on the feeder at the large end of the antenna.

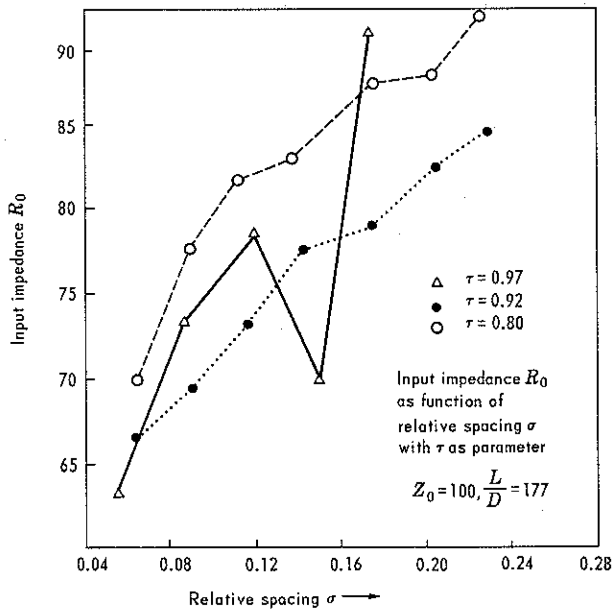

Fig. 7.14 Mean input impedance level of log-periodic dipole array as a function of relative spacing σ (σ is ratio of element spacing to twice the length of the next larger element).

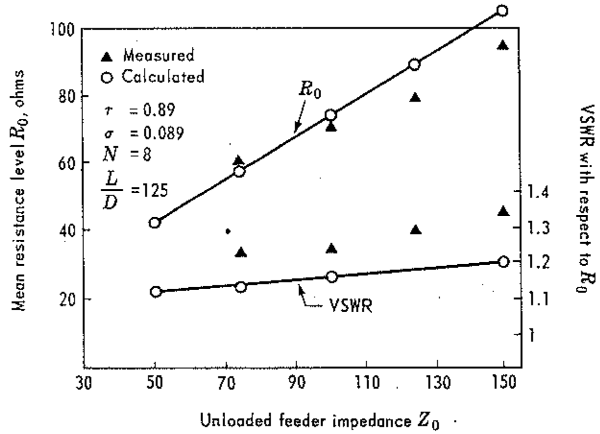

Fig. 7.15 Input impedance of log-periodic dipole array as a function of the characteristic impedance of the unloaded feeder line.

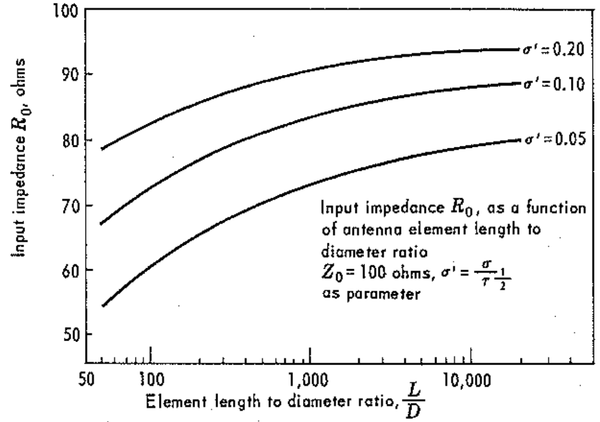

Fig. 7.16 Mean input impedance level of log-periodic dipole arrays as function of element length-to-diameter ratio.

Carrel has prepared curves that are invaluable in the design of log-periodic dipole antennas. As pointed out earlier, the design parameters are τ (the ratio of distances of successive elements from the apex), α (the half-angle subtended at the apex), and σ (the ratio of the element spacing to twice the next-larger element length). Of course, only two of these variables are independent. Figure 7.17 shows how the directive gain varies with σ and τ. The data are displayed in the form of directivity contours. It will be noted that gains in the range from 7.5 to 12 db over isotropic can be obtained with the proper choice of parameters. For a given σ, τ, and element length-to-diameter ratio, the input impedance depends on the characteristic impedance of the parallel-rod feeder line. Thus an antenna may be designed so as to give a specified directivity, and thereafter the input impedance can be almost independently adjusted to a required level. The bandwidth and the specific frequency range required determine the absolute size and arrangement of the structure.

As an example, let us illustrate the design of a log-periodic dipole array for use between 20 and 60 MHz. The antenna is to have a gain of 10 db and an input impedance of 75 ohms. From Fig. 7.17, along the optimum σ line, we find $\tau = 0.917$, and $\sigma = 0.172$. From the geometry, the angle α is

$$\alpha = \tan^{-1} \frac{1 - \tau}{4\sigma}$$

or about 7°. The bandwidth required is 3:1, and roughly speaking the

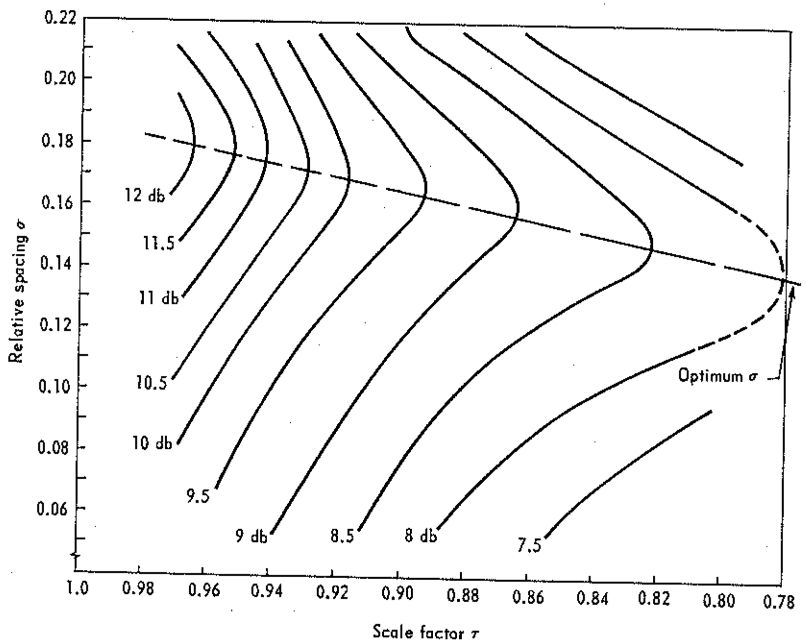

Fig. 7.17 Contours of constant directivity versus τ and σ, for log-periodic dipole arrays.

longest element should be a half wavelength at the lowest frequency. However, the size of the active region depends on the specific design. To incorporate this feature into the design planning, Carrel has introduced and calculated a factor which in essence gives the required structure size to achieve a desired bandwidth. Essentially, one designs for a slightly larger bandwidth than would be required if the active region were of negligible length along the structure. This larger bandwidth B_s is related to the actual required bandwidth B by a relation $B_s = BB_{ar}$, where the factor B_{ar} is tabulated as a function of σ and τ. Carrel calls this quantity the bandwidth of the active region. A nomograph for B_{ar} is presented in Fig. 7.18. In our example, since α is about 7°, B_{ar} is 1.54; therefore $B_s = 3 \cdot 1.54 = 4.56$. Taking the longest element to be a half wavelength at the low-frequency end of the band B_s, it follows from the geometry of the structure that the distance L between the shortest element and the longest element is

$$\frac{L}{\lambda_{max}} = \frac{1}{4}\left(1 - \frac{1}{B_s}\right) \cot \alpha \qquad (= 1.7)$$

Again, from the geometry and the relation $x_N/x_1 = \tau^{N-1}$, the number of elements is given by the equation

$$N = 1 + \frac{\log B_s}{\log (1/\tau)} \qquad (= 18)$$

The directivity is a slight function of length-to-diameter ratio. In

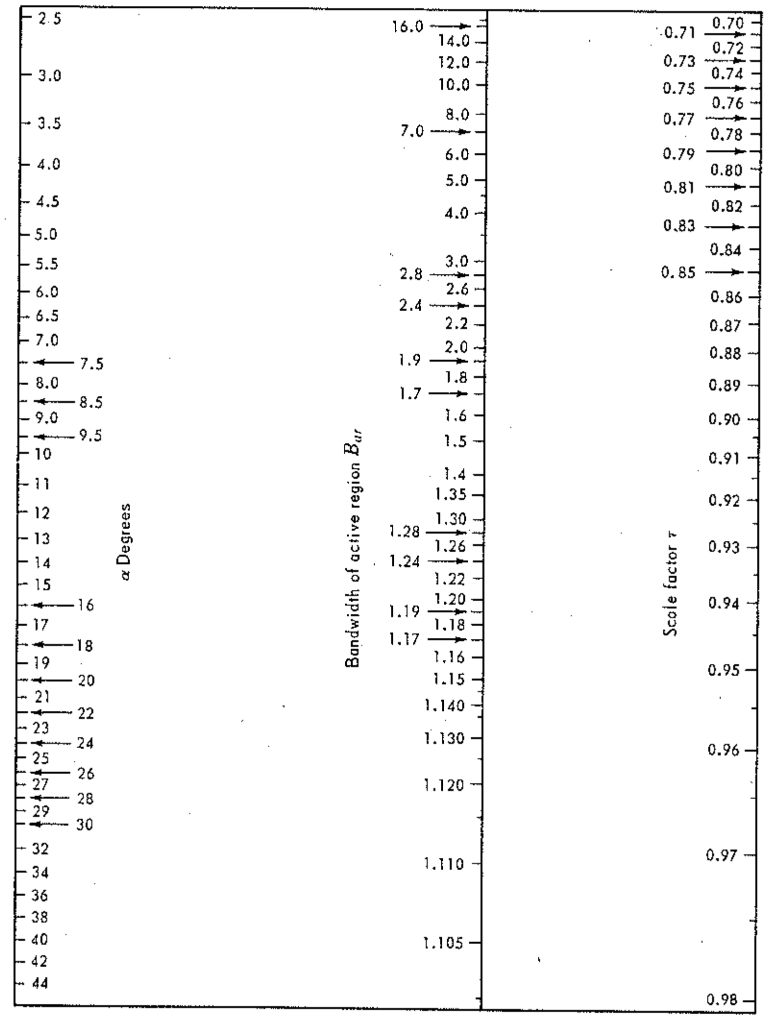

Fig. 7.18 Nomograph, $B_{ar} = 1.1 + 7.7(1 - \tau)^2 \cot \alpha$.

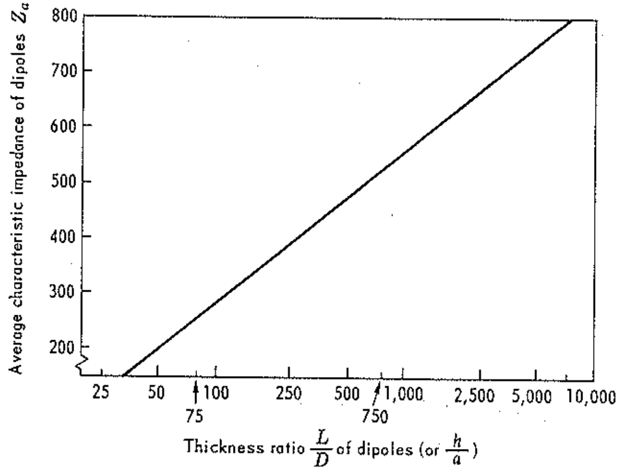

Fig. 7.19 Average characteristic impedance of dipoles Z_{al}, as a function of length-to-diameter ratio.

our particular case, to reach the lower end of the frequency band, the largest elements would have a half-length of about 14 ft. For strength, the elements might be constructed of 1½-in. aluminum tubing. If this were the case, the length-to-diameter ratio would be about 200; therefore no correction to Carrel's main curves would be required.

Next we must design for the required input impedance R_0. This quantity depends on Z_0, σ, and the length-to-diameter ratio of the elements. To bring in the effect of the latter, it is convenient to work in terms of an average characteristic impedance for the elements. An approximate formula for the average characteristic impedance of the elements is

$$Z_a = 120\left(\ln \frac{h}{a} - 2.25\right)$$

where h/a is the half-length-to-radius ratio. A graph for Z_a is given in Fig. 7.19. For our example, the average characteristic impedance is something like 360 ohms. The loading produced by such elements depends on the spacing. The effect is indicated quantitatively in the graph, Fig. 7.20. Therein, the quantity σ' is a mean relative spacing, $\sigma' = \sigma/(\tau)^{1/2} = 0.18$. Thus, since $Z_a/R_0 = {}^{360}\!/_{75} = 4.8$, the graph indicates that $Z_0/R_0 = 1.1$; that is, we should design a parallel-rod feeder such that if unloaded it would have a characteristic impedance of $Z_0 = 1.1 \times 75 = 83$ ohms. It will be recalled that the characteristic impedance of a parallel-rod line is given by

$$Z_0 = 120 \cosh^{-1} \frac{s}{D}$$

where s is the center-to-center spacing, and D is the diameter of the rod. This completes the design, except for the details of picking the specific lengths and tube sizes.

If the diameter of the largest element were selected to be 1½ in., then this implies that the feeder-rod diameter should also be at least 1½ in. in diameter. This sets the center-to-center spacing on the feeder line as follows:

$$s = D \cosh \frac{Z_0}{120} = 1.87 \text{ in.}$$

The length of the parallel-rod line between the longest and the shortest elements is $1.6\lambda_{max}$, or about 90 ft, a rather long structure. The lengths and positions of the other elements follow from τ and σ. An effort should be made to select tubing for the various elements in such a way that the length-to-diameter ratio is nearly the same for all elements. Sometimes it is necessary or desirable to scale down the size of the parallel-rod feeder line in the region of the smaller elements.

The design presented in the foregoing example is by no means optimum. In practice, several such designs would be worked out in detail and the results studied, in order to minimize the antenna length, or the

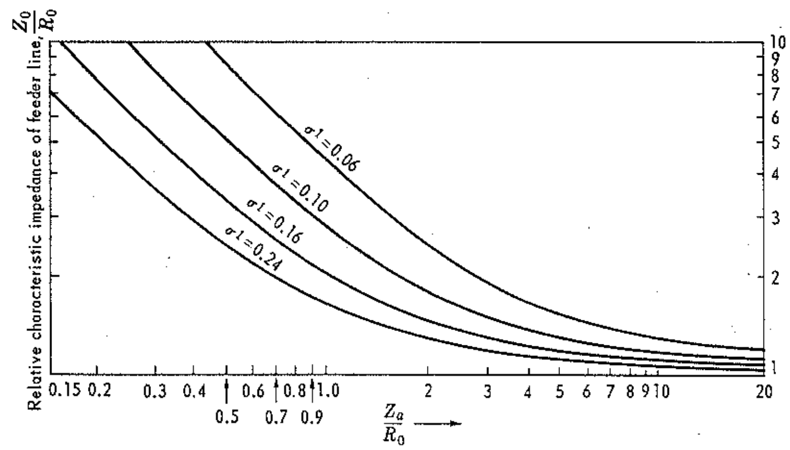

Fig. 7.20 Relative characteristic impedance of feeder line as function of relative characteristic impedance of dipole elements loading the line, Z_a/R_0.

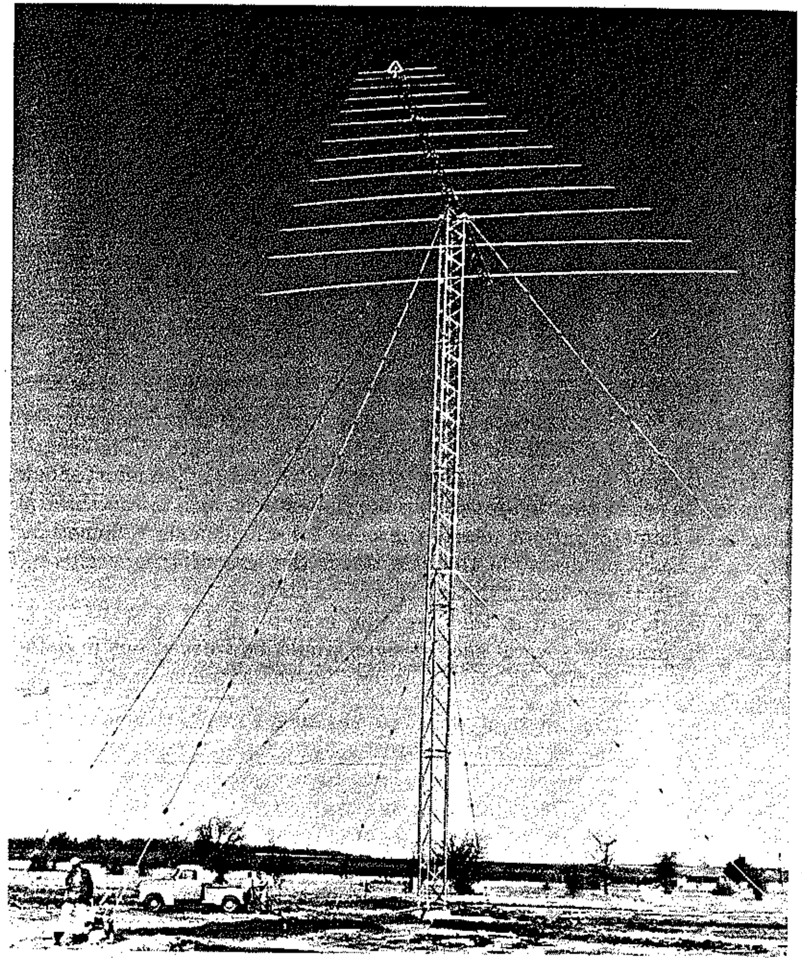

Fig. 7.21 Collins Radio Company 237B series log-periodic dipole array.

number of elements, or to optimize the design in some other respect. As might be expected, the higher gains require the longer structures. A photograph of a commercial version of a log-periodic dipole array, for use in the HF band, is shown in Fig. 7.21.

We have now traced the development of the log-periodic concept and have studied in some detail the log-periodic dipole array as a concrete example of this concept. But a little thought reveals that there is an

infinite variety of log-periodic structures. It is therefore worthwhile to summarize by discussing the log-periodic concept in general, without regard to any specific structure. The basic idea is that of electromagnetic modeling or scaling. If all dimensions of a lossless antenna are changed by a factor of τ, then the antenna performance is identical at a frequency $1/\tau$. Now consider the class of antennas made up of an infinite number of "cells," which differ from each other only in size; in particular, each differs from its neighbor only by a constant scale factor τ, as indicated in Fig. 7.22. Structures of this class may be called self-similar, because they possess the unique property of transforming into themselves under a uniform expansion by τ or an integral power of τ. This means that the performance is identical for all frequencies related by $f = f_0\tau^p$, $p = \pm 1, \pm 2, \ldots$. If voltages and currents are defined at corresponding points in the cells of the structure, these must satisfy the relationships

$$V_n(f) = V_{n+1}(\tau f)$$
$$I_n(f) = I_{n+1}(\tau f)$$

There are essentially no restrictions on the internal composition of the cells as long as they scale with frequency; they may include scaled sources and sinks. However, most work to date has been confined to a single

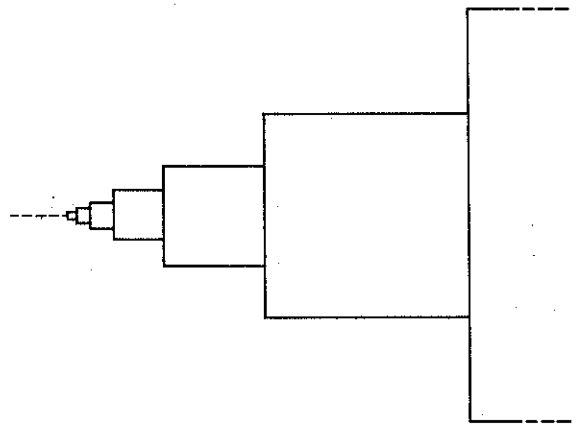

Fig. 7.22 A generalized set of self-similar cells in a structure, selected so that the dimensions of one are τ times the size of the next larger. This is a generalized log-periodic structure.

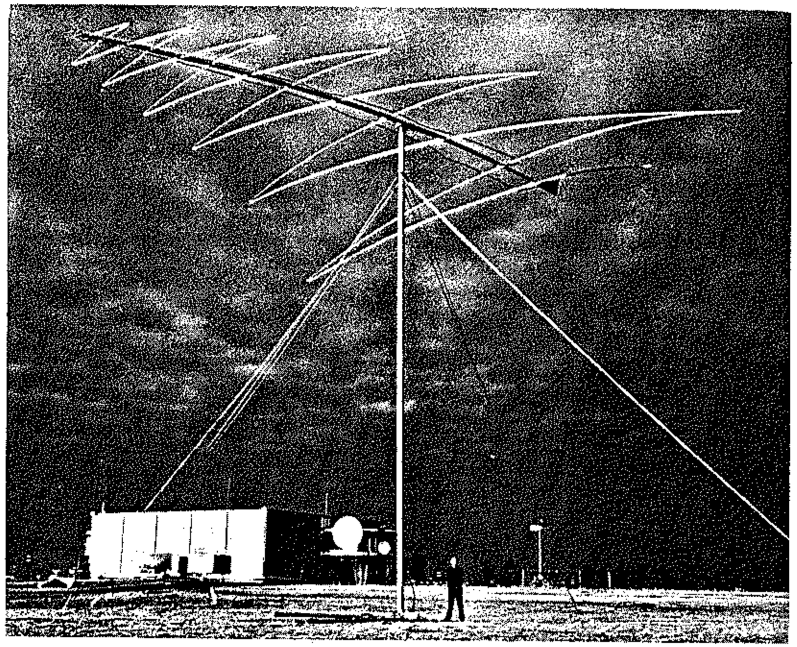

Fig. 7.23 Rotatable log-periodic antenna for HF band. Shunt-fed elements. (Collins Radio Company 637B.)

generator at a single location, which must be at the small end in a finite structure.

The practical structure must be finite in size; it must start with some smallest cell and end with some largest cell. Yet at the frequencies within the operating band, the structure must "look" infinite. This implies that at the lower frequencies the electrically small cells must behave as transmission networks. If the cells are electrically small enough, the structure is a *circuit*, which is to say that whether the generator is located in one of these small cells or another is of no significance. It also implies that the larger cells must not be excited, so that their presence or absence will make no difference in the performance within the operating band of frequencies. If these restrictions can be met, the result will be a structure of the log-periodic type, independent of the details of the individual cells. The further art in making a broadband antenna is to choose the cells and the scaling parameters so that the performance between frequencies $f_0\tau^p$ is satisfactorily constant. Figure

7.23 is a photograph of a log-periodic antenna having a different type of cell structure from that discussed earlier. In this structure, the radiators are shunt-fed through circuit elements. The results of near-field measurements performed on a structure of this type, excited, however, from the small end in a series fashion, are available in the literature.[1]

Problem 7.1 Design a log-periodic dipole antenna to operate over the frequency band from 50 to 250 MHz. To have a specific goal in mind, suppose that the antenna is to cover the VHF TV bands plus the FM bands.

After carrying out the design in a straightforward way, consider what might be done to make a shorter structure, in view of the fact that continuous coverage is not required all the way from 50 to 250 MHz in the specific application.

[1] R. L. Bell, C. T. Elfving, and R. Franks, *IRE Trans.*, **AP-8**:559 (November, 1960).