Ian Hickman introduces aspects of passive filter design that are important yet frequently omitted from standard descriptions.

Filter VARIATIONS

Many applications call for the filtering of signals, to pass those that are wanted and to block those that are outside the desired passband. Sometimes digital filtering is appropriate, especially if the signals are in digital form already, but oftentimes analog filters suffice – indeed are the only choice at rf. At lower frequencies, where inductors would be bulky, expensive and of low Q active filters are the usual choice Some of these are documented in every text book, but there are some useful variations upon the them which are less well known This article explores one or two of these.

A basic active filter

Probably the best known active filter is the Sallen and Key second order circuit, the lowpass version of which is shown in **Figure 1** Interchanging the Cs and Rs gives a highpass version There has been considerable discussion recently of its demerits, both in regard to noise and distortion, from Dr D Ryder and others in the Letters section of this magazine, see the November 1995 to April 1996 issues inclusive. But for many purposes it will prove adequate having the minor advantage of very simple design equations. Moreover, the circuit is canonic - it uses just two resistors and two capacitors to provide its two-pole response

Being a second order circuit, at very high frequencies the response falls away forever at 12dB per octave, at least with an ideal opamp In practice, opamp output impedance rises at high frequencies due to the fall in its open loop gain, resulting in the attenuation curve levelling out or even reversing In the maximally flat amplitude response design, at frequencies above the cutoff frequency, the response approaches 12dB/octave asymptotically from below At dc and well below the cutoff frequency the response is flat being 0dB (unity gain) again a value the response approaches asymptotically from below The corner formed by the crossing of these two asymptotes is often called, naturally enough, the 'corner frequency'. The corner or cutoff frequency f_0 is given by $f_0=1/(2\pi\sqrt{C_1C_2R_1R_2})$ where usually $R_1=R_2$.

The dissipation factor $D=1/Q$ where $Q=0.5\sqrt{(C_1/C_2)}$ and for a maximally flat amplitude (Butterworth) design $D=1.414$, so $C_1=2C_2$. The Butterworth design exhibits no peak and is just 3dB down (ie $V_{out}/V_{in}=0.707$, or equal to Q) at the corner frequency. If $C_1>2C_2$, then there is a passband peak in the response below the corner frequency being more pronounced and moving nearer the corner frequency as the ratio is made larger This permits the design of filters with four or six poles, or of even higher order, consisting of several such stages, all with the same corner frequency but each with the appropriate value of Q.

It is easy to see that the low frequency gain is unity by simply removing the capacitors from Figure 1 for at very low frequencies their reactance becomes so high compared to R_1, R_2 that they might as well simply not be there. At a very high frequency, way beyond cutoff C_2 acts as a near short at the non-inverting (NI) input of the opamp resulting in the lower plate of C_1 being held almost at ground As C_1 is usually greater than C_2, it acts in conjunction with R_1 as a passive lowpass circuit well into its stopband resulting in even further attenuation of the input At twice this frequency, both of these mechanisms will result in a halving of the signal which thus falls to a quarter of the previous value, ie the roll-off rate is 20log(1/4) or –12db/octave But what about that peak in the passband, assuming there is one?

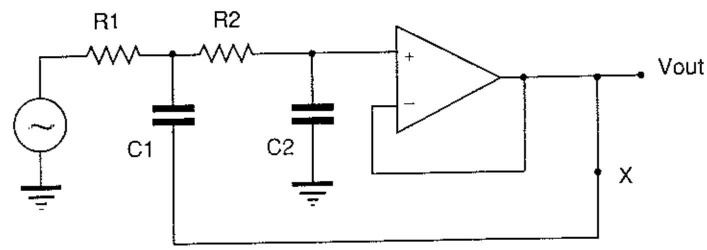


Fig. 1 The Sallen and Key second order lowpass active filter. Cut-off 'corner' frequency is given by $f_o=1/(2\pi C_1C_2R_1R_2)$ and $Q=0.5\sqrt{(C_1/C_2)}$ and dissipation D=1/Q. For a maximally flat amplitude (Butterworth) design, D=1.414, so $C_1=2C_2$. The Butterworth design exhibits no peak, and is just 3dB down at the corner frequency.

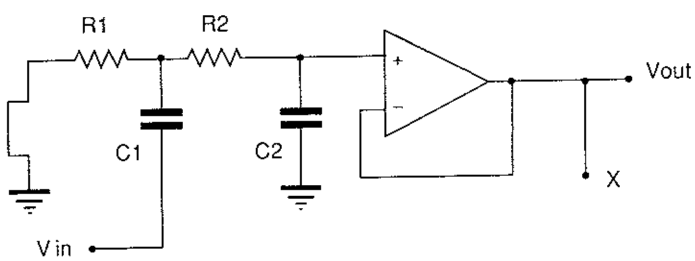


Fig. 2 Breaking the loop and opening it out helps to understand the circuit action (see text).

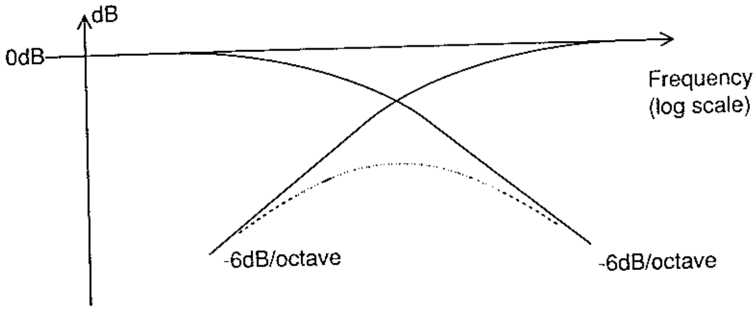


Fig 3. Cascaded lowpass and highpass CR responses, and their resultant, (dotted).

The best way to approach this is to break the loop at point X, in Figure 1 and consider what happens to a signal V'_{in}, going round the loop, having removed the original V_{in} Note that as the source in Figure 1 is assumed to have zero internal resistance, it has been replaced by a short circuit in **Fig. 2**. To V'_{in}, C_1 with R_1 now forms a passive lead circuit – highpass or bass-cut The resultant voltage across R_1 is applied to C_2, R_2, a passive lag circuit – lowpass or top-cut.

Each of these responses exhibits a 6dB/octave rolloff in the stopband, as shown in **Fig. 3** Thus the voltage reaching the NI input of the opamp at any frequency will be roughly the sum of the attenuation of each CR section (actually rather more, as C_2R_2 loads the output of the C_1R_1 section), as indicated by the dotted line in Figure 3. At the frequency where the highpass and lowpass curves cross, the attenuation is a minimum and the phase shift is zero since the lag of one section cancels the lead of the other

If C_1 is now made very large, the bass cut will only appear at very low frequencies – the highpass curve in Figure 3 will shift bodily to the left If in addition, C_1 is made very small, the top cut will appear only at very high frequencies – the lowpass curve will shift bodily to the right. Thus the curves will cross while each still contributes very little attenuation, so the peak of the dotted curve will not be much below 0dB, unity gain. Consequently, at this frequency the voltage at X is almost as large as V_{in}, and in phase with it The circuit can almost supply its own input, and if disturbed in any way will respond by ringing at the frequency of the dotted peak where the loop phase shift is zero

But however large the ratio C_1/C_2, there must always be some attenuation, however small, between V'_{in} and the opamp's NI input, so the circuit cannot oscillate, although it can exhibit a large peak in its response, around the corner frequency In fact, if the peak is large enough, the response above the corner frequency will approach the –12dB/octave asymptote from above, and below the corner frequency will likewise approach the flat 0dB asymptote from above

Variations on a theme

The cutoff rate can be increased from 12dB/octave to 18dB/octave by the addition of just two components; a series R and a shunt C to ground between V_{in} and R_1 And such a third order section can be cascaded with other second order section(s) to make filters with 5 7, 9 poles etc Normalised capacitor values for filters from 2 to 10 poles for various response types (Butterworth, Chebychev with various passband ripple-depths, Bessel etc) have been published in Refs 1 and 2, and in many other publications as well However, these tables assume R_1=R_2 (= the extra series resistor in a third order section), with the Q being set by the ratio of the capacitor values This results in a requirement for non-standard values of capacitor, which is expensive if they are specially procured, or inconvenient if made up by parallelling smaller values

While equal value resistors is optimum, minor variations can be accommodated without difficulty, and this can ease the capacitor requirements Ref 3 gives tables for the three resistors and three capacitors used in a third order section with the capacitors selected from the standard E3 series (1 0, 2 2, 4 7) and the resistors from the E24 series for both Butterworth and Bessel (maximally flat delay) designs

The Kundert filter

The formula for the Q of the Sallen and Key filter is $Q=0.5\sqrt{(C_1/C_2)}$ so given the square root sign and the 0 5 as well, one finishes up with rather extreme ratios of C_1 to C_2, if a high Q is needed, as it will be in a high order Chebychev filter. In this case, the Kundert circuit of **Fig. 4** may provide the answer The additional opamp buffers the second CR from the first so that the attenuation at any frequency represented by the dotted curve in Figure 3 is now exactly equal to the sum of the other two curves. Removing the loading of C_2R_2 from C_1R_1 removes the 0.5 from the formula, which is now $Q=\sqrt{(C_1/C_2)}$ – assuming R_1=R_2. And due to the square root sign, the required ratio of C_1 to C_2 for any desired value of Q is reduced by a factor of four compared to the Sallen and Key version

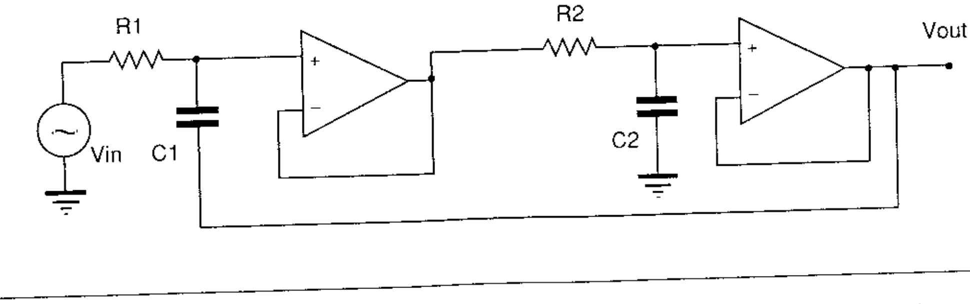
A further advantage of this circuit is the complete freedom of choice of components Instead of making R_1=R_2 and setting the Q by the ratio of C_1 to C_2 the capacitors may be made equal and the Q set by the ratio of R_1 to R_2 or both Cs and Rs may differ, the Q being set by the ratio of C_1R_1 to C_2R_2 Given that dual opamps are available in the same 8 pin DIL package as single opamps, the Kundert version of the Sallen and Key filter, with its greater freedom of choice of component values, can come in very handy for the highest Q stage in a high order filter

The equal C filter

In addition to filtering to remove components outside the wanted passband, signals also frequently need amplification. The basic Sallen and Key circuit only provides unity gain, and with this arrangement, equal resistors are optimum For, due to the loading of the second stage on the first if R_2 is increased to reduce the loading, then C_2 will have to be even smaller, whilst if R_2 is decreased to permit a larger value of C_2, the loading on C_1R_1 increases

Where additional signal amplification is needed there is no reason why some of this should not be provided within a filtering stage and **Fig. 5** shows such a circuit Clearly the dc and low frequency gain is given by (RA+RB)/RB A convenience of this circuit is that the ratio RA to RB can be chosen to give whatever gain is required (within reason), with C_1, C_2, R_1 and R_2 chosen to give the required corner frequency and Q An analysis of this most general form of the

Fig 4. The Kundert filter, a variant of the Sallen and Key, has some advantages.



circuit can be found in Ref 4 If there were a buffer stage between R_1 and R_2 as in Figure 4, and the two CR products were equal, then at a frequency of 1/(2πCR) there would be exactly 3dB attenuation round the loop due to each CR.

So if RA were to equal RB, giving 6dB gain in the opamp stage, there would be no net attenuation round the loop and the Q would equal infinity – you have an oscillator. Without the buffer opamp, the sums are a little more complicated due to the second CR loading the first But the sums have all been done, and the normalised values for R_1 and R_2 (values in ohms for a cutoff frequency of 1/2πHz, assuming C=1F) are given in Ref 5 for filters of 1 to 9 poles, in Butterworth, Bessel and 0.1dB- 0.5dB- and 1dB-Chebychev designs.

For odd numbers of poles this reference includes an opamp buffered single pole passive CR, rather than a three pole version of the Sallen and Key filter, as one of the stages. To convert to a cutoff frequency of, say, 1kHz regard the ohms figures in the tables as Mohms and the capacitors as 1μF Now divide the resistor values by 2000π As the values are still not convenient, scale the capacitors in a given section down by say 100 or any other convenient value, and the resistors up by the same factor

Reference 5 also gives the noise bandwidth of each filter type. The noise bandwidth of a given filter is the bandwidth of a fictional ideal brick wall sided filter which fed with wideband white noise, passes as much noise power as the given filter Ref. 5 also gives, for the Chebychev types, the 3dB bandwidth Note that for a Chebychev filter, this is not the same as the specified bandwidth (unless the ripple depth is itself 3dB) For a Chebychev filter the bandwidth quoted is the ripple bandwidth; e g. for a 0.5dB ripple lowpass filter, the bandwidth is the highest frequency at which the attenuation is 0.5dB, beyond which it descends into the stopband passing through –3dB at a somewhat higher frequency

Other variants

In the Sallen and Key filter, the signal appears at both inputs of the opamp. There is thus a common mode component at the input and this can lead to distortion, due to 'common mode failure', which, though small, may be unacceptable in critical applications Also, as already mentioned, the ultimate attenuation in the stopband will often be limited by another non-ideal aspect of practical opamps - rising output impedance at high frequencies, due to the reduced gain within the local NFB loop back to the opamp's inverting terminal Both of these possibilities are avoided by a different circuit configuration, shown in its lowpass form, in **Figure 6a)**

This is variously known as the infinite gain multiple feedback filter, or the Rausch filter and it has the opamp's NI terminal firmly anchored to ground – good for avoiding common mode failure distortion. Another plus point is that at very high frequencies, C_1 short circuits the signal to ground, while C_2 shorts the opamp's output to its inverting input – good for maintaining high attenuation at the very highest frequencies The design equations and tabulated component values are available in published sources; the filter is well known and is shown here just as a stepping stone to a less well known filter section, the SAB (single active biquad) with finite zero.

In some filtering applications, the main requirement is for a very fast rate of cutoff, the resultant wild variations in group delay not being important The Chebychev design provides a faster cut off than the Butterworth, the more so, the greater ripple depth that can be tolerated in the passband But the attenuation curve is monotonic, it just keeps on going down at (6n)dB/octave, where n is the order of the filter (the number of poles), not reaching infinite attenuation until infinite frequency

A faster cutoff still can be achieved by a filter incorporating one or more 'finite zeros' frequencies in the stop band at

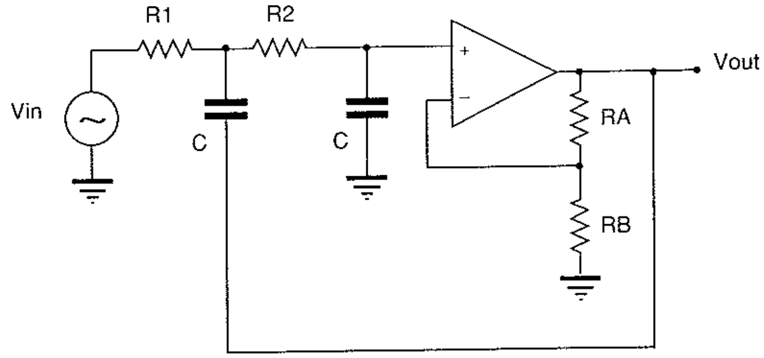


Fig. 5 The equal C version of the Sallen and Key circuit.

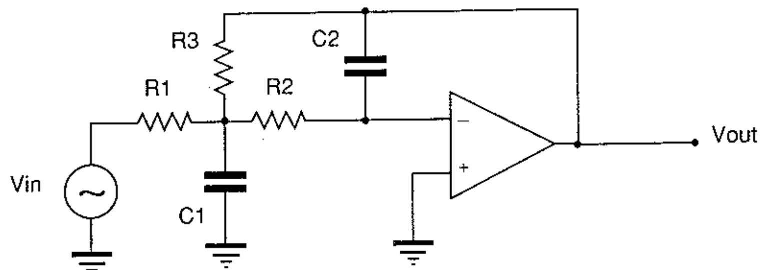


Fig. 6a) The mixed feedback or 'Rausch' filter – lowpass version.

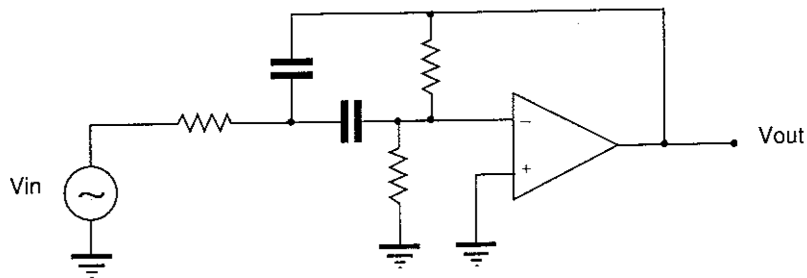


Fig 6b). The mixed feedback or 'Rausch' filter – bandpass version

which the response exhibits a notch. In a design with several such notches, they can be strategically placed so that the attenuation curve bulges back up in between them to the same height each time Such a filter with equal depth ripples in the passband (like a Chebychev) but additionally with equal returns between notches in the stop band is known as an 'elliptic' or 'Caur' filter

In a multipole elliptic filter, each second order section is designed to provide a notch but beyond the notch the attenuation returns to a steady finite value, maintained up to infinite frequency. The nearer the notch to the cutoff frequency, the higher the level to which the attenuation will eventually return above the notch frequency.

So for the highest cutoff rates, while still maintaining a large attenuation beyond the first notch, a large number of poles is necessary It is common practice to include a single pole (eg an opamp buffered passive CR lag) to ensue that, beyond the highest frequency notch the response dies away to infinite attenuation at infinite frequency albeit at a leisurely –6dB/octave

The elliptic filter

The building blocks for an elliptic lowpass filter consist of second order lowpass sections of varying Q each exhibiting a notch at an appropriate frequency above the cutoff frequency.

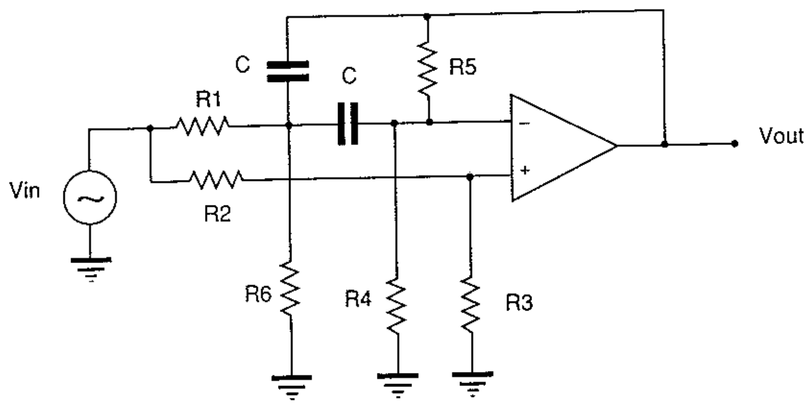


Fig. 7. The SAB circuit, with finite zero (or notch, above the passband)

A number of designs for such a section have appeared, based on the twin-tee circuit, but they are complex, using many components, and hence difficult to adjust. An alternative is provided by the SAB section mentioned earlier. This can be approached via the Rausch bandpass filter, which can be seen in **Figure 6b)** to be a variant on the Rausch lowpass design of Figure 6a). Clearly, due to the capacitive coupling the circuit has infinite attenuation at 0Hz, and at infinite frequency, the capacitors effectively short the opamp's inverting input to its output, setting the gain to zero. Either side of the peak response, the gain falls off at 6dB per octave, the centre frequency Q being set by the component values. If the Q is high, the centre frequency gain will be well in excess of unity.

Figure 7 shows the same circuit with three extra resistors (R_2, R_3 and R_6) added. Note that an attenuated version of the input signal is now fed to the NI input of the opamp via R_2, R_3. Consequently, the circuit will now provide finite gain down to 0Hz; it has been converted into a lowpass section, although if the Q is high there will still be a gain peak. If the ratio of R_5 to R_4 is made the same as R_2 to R_3, then the gain of the opamp is set to the same as the attenuation suffered by the signal at its NI terminal, so the overall 0Hz gain is unity.

If the other components are correctly chosen, the peak will still be there, but at some higher frequency, the signal at the opamp's inverting input will be identical in phase and amplitude to that at the NI input. The components thus form a bridge which is balanced at that frequency, resulting in zero output from the opamp, ie a notch.

Figure 8 shows a five pole elliptic filter using SAB sections, with a 0.28dB passband ripple, a –3dB point at about 3kHz and an attenuation of 54dB at 4.5kHz and above. The design equations for elliptic filters using SAB sections are given in Ref 6. The design equations make use of the tabulated values of normalised pole and zero values given in Ref 7.

Some other filter types

Simple notch filters – where the gain is unity everywhere either side of the notch – can be very useful, eg for suppressing 50Hz or 60Hz hum in measurement systems. The passive TWIN TEE notch is well known, and can be sharpened up in an active circuit so that the gain is constant, say, below 45Hz and above 55Hz. However, it is inconvenient for tuning, due to the use of no less than six components. An ingenious alternative[8] provides a design with limited notch depth, but compensating advantages. A notch depth of 20dB is easily achieved, and the filter can be fine tuned by means of a single pot. The frequency adjustment is independent of attenuation and bandwidth.

Finally, a word on linear phase (constant group delay) filters. These are easily implemented in digital form, FIR filters being inherently linear phase. But most analog filter types, including Butterworth, Chebychev and elliptic are anything but linear phase. Consequently, when passing pulse waveforms, considerable ringing is experienced on the edges, especially with high order filters, even of the Butterworth variety. The linear phase Bessel design can be used, but this gives only a very gradual transition from pass- to stop-band, even for quite high orders. However, a fact that is not widely known is that it is possible to design true linear phase filters in analog technology, both bandpass[9] and lowpass[10]. These can use passive components, or – as in Reference 10 – active circuitry. ■

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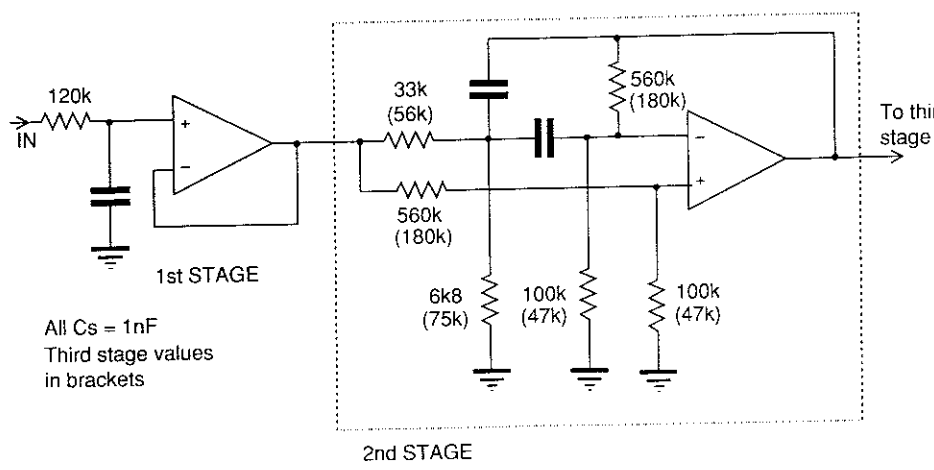

Fig. 8. A five-pole elliptic filter with 0.28dB passband ripple and an attenuation of 54dB at 1.65 times the cutoff frequency and upwards. The –3dB point is 3kHz, approx. All capacitors C=1nF, simply scale C for other cutoff frequencies.