AN ETERNAL ENIGMA:

THE APPLICABLE AND CONSTRUCTABLE FICTIONS OF ELECTRONICS

by M.C. Soper, MA

SOME methods used in electronics are based on models that cannot, in fact, be the case. For example, currents flow continuously and yet consist of the flow of objects: discrete objects, called electrons. Waveforms are commonly denoted by $e^{j\omega t}$, yet, $e^{j\omega t} = \cos\omega t + j\sin\omega t$, where j is defined by $j^2 = -1$, which is not true for any real number: j is imaginary.

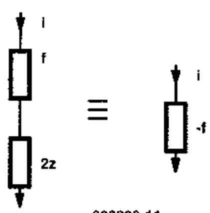
Fictional models like this build the theory on which circuit calculations are based. These fictions are all mathematical and are used to practical effect in calculations. Recently, fictional circuit elements have also been used, like the *nullor* and the *nullator*. Other fictional elements can be joined to the system for ease of circuit calculation; for example, *negative time delays* and *recursive components*.

Can we build it?

Evidently, theoretical use is made of circuit elements that either cannot be made or cannot be manufactured at our present state of knowledge, but may become possible at some future time. We can have physically difficult things to make or theoretically different things to make: a theoretically difficult thing to make is a single, infinitely fast, perfect active switch. A practically difficult thing to make is a minute, large-value, passive inductor. Our fictional elements can:
(a) make a theoretical construction of a practically difficult (or even theoretically difficult) elements possible;
(b) simplify complex calculations.

Why this option?

This approach may be preferred since diagrammatic rather than mathematical methods can be used; that is, a simple understanding of a diagram together with fairly basic computational skills can replace very complex techniques in some cases. One more reason for preferring the use of fictitional elements is that new circuit elements of a theoretical kind can be specified easily, whereas to describe them in other ways may well be lengthy. For example:



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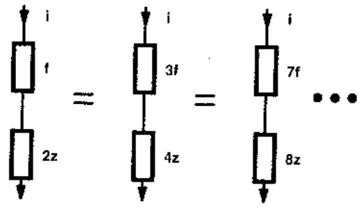
Since these are equivalent,

$$2z + f = -f,$$

where f is thought of like impedance, so that

$$f = -z$$

Thus, recursively we have defined a negative impedance. The recursion in this case is very simple. Note, however, that



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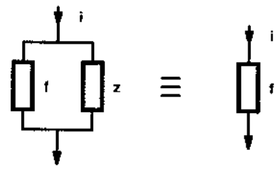
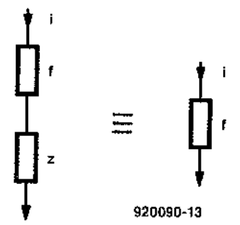
As another example,

for any z



defines a short circuit

and



defines an open circuit

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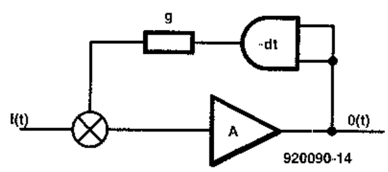
We can also introduce another fictional topic: instead of time delays, time increments. Obviously, these cannot exist, because a signal would be output before it had been input, but



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has this equation:

$$0(t) = A[I(t) + g\{o(t-dt)\}]$$

Assuming that A and g are reversible and linear:

$$A^{-1}[0(t)] = I(t) + g[0(t-dt)]$$

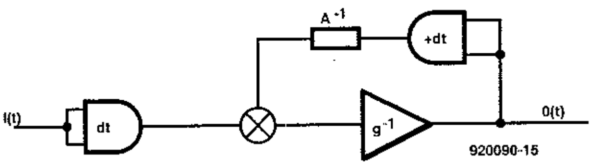
Since this is true at any time,

$$A^{-1}[0(t+dt)] = I(t+dt) + g[0(t)]$$

$$\therefore g[0(t)] = -I(t+dt) + A^{-1}[0(t+dt)]$$

$$\therefore\ 0(t) = g^{-1}\{A^{-1}[0(t+dt)] - I(t+dt)\} \quad [1]$$

This is a present output with respect to future events; it may be paradoxical, but it has been solved by Richard Feynman Equation [1] may be obtained from this circuit, where the fictional positive time increment (pti) elements have been included



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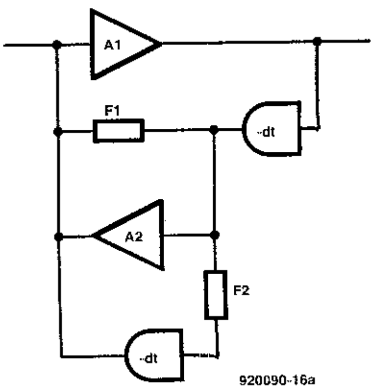
At this stage, the circuit looks merely eccentric, but consider how this type of transition may be used to reduce the complexity of circuits with active elements and delays in the feedback loop Note also that we can write:

$$0(t-dt) = g^{-1}A^{-1}[0(t)] - g^{-1}I(t)$$

Releasing practical constraints

Consider



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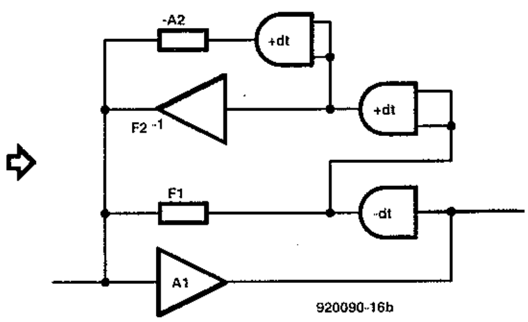

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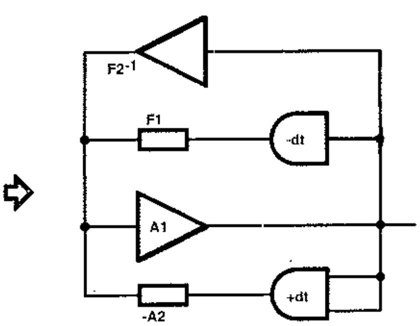

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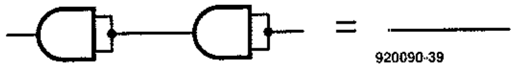
This can be simplified by introducing fictional ptis. The first step is from Equation [1]; the second step is from the fact that



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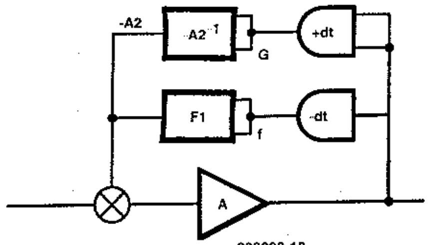
and the third step is from standard and known properties of amplifiers in gfeedback loops. We have, therefore, this remarkable simplification, which is easy to calculate:



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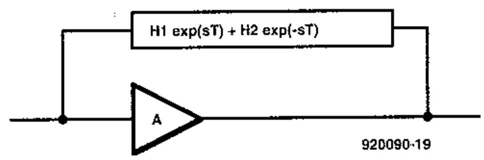
Let $A = A_1/(1+A_1F_2^{-1})$;

then

$$0(t) = A\{I(t) + f[0(t-dt)] + g[0(t+dt)]\};$$

or,



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Transfer function $A_0 = 1/[be^{-sT} - ae^{sT} + (1/A)]$.

To check further the equivalence of Eq [1], here is a negative feedback form with linear amplifying and feedback elements:

$$\begin{aligned} 0(t) &= A\{I(t) - f[0(t-dt)]\} \\ &= AI - f\{I(t-dt) - f[0(t-2dt)]\} \\ &= A\textstyle\sum_r A^r f^r[(-1)^r I(t-rdt)], \end{aligned}$$

which, if I is constant,

$$= I_c A \textstyle\sum_r A^r f^r (-1)^r$$

$$= AI_c/(1+Af)$$

If $i = I_0\cos(2\pi ft)$, then, with $B=f$ (feedback) and $AB<1$, we get

$$0(t) = A\textstyle\sum_r A^r f^r (-1)^r \cos(2\pi ft - 2\pi frdt)$$

$$= \frac{A^2 B\cos[2\pi f(t-dt)] + A\cos 2\pi ft}{A^2B^2 + 2AB\cos(2\pi fdt) + 1}$$

Note that this can also be written without reference to time as the equivalence of operators:

$$0() = \frac{A[AB\cos(2\pi fdt) + 1]\cos[2\pi f()] + A^2 B\sin(2\pi fdt)\sin[2\pi f()]}{1 + 2AB\cos(2\pi fdt) + A^2B^2}$$

Consider the reorganized equation:

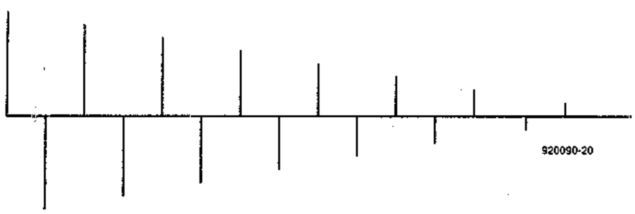
$$O(t)=B^{-1}[I(t+\mathrm{d}t)+(-A^{-1})O(t+\mathrm{d}t)]$$

$$=B^{-1}\sum_r(-A^{-1}B^{-1})^r\cos[2\pi f(t+\mathrm{d}t)(1+r)]$$

$$=\frac{B^{-1}\cos(2\pi ft)+B^{-2}A^{-1}\cos[2\pi f(t-\mathrm{d}t)]}{1+2A^{-1}B^{-1}\cos(2\pi f\mathrm{d}t)+A^{-2}B^{-2}}$$

$$=\frac{A^2B\cos[2\pi f(t-\mathrm{d}t)]+A\cos(2\pi ft)}{1+2AB\cos(2\pi f\mathrm{d}t)+A^2B^2}$$

As expected, the reorganized equation, which gives the present output in terms of future inputs and outputs, is just as valid: this is because the signal is one unvarying sine wave and thus conveys no information None the less, all wave forms can be made out of sine waves of a range of frequencies summed Thus this conclusion holds in general The main intuitive problem is indicated by the response of a delayed negative feedback linear amplifier as shown



Our analogue for this is a positive feedback amplifier with a pti of the same value, and the input passing through an input stage consisting of a pti before the feedback sum junction is reached. This will have the same response and diagram as above; but the problem for the intuition is that one would expect that a 'pulse-to-come' would create an infinite series of pre-pulses, not an an infinite series of post-pulses The reason for this is that each of the post-pulses consists of the superimposed sine waves of appropriate frequency and phase. *Before* the stimulus pulse arrives, these sine waves cancel completely, but this cancellation is marred by the arrival of the stimulus pulse, which results in the chain of post-pulses normally seen

From Fourier's Transform Theorem, linearity and the two identical sine wave formulae just shown, we know the outputs will be identical to those illustrated in the spike diagram

Many people will feel very uneasy about including an impossibility in a circuit diagram, since this is something that cannot possibly be made But, a perfect opamp can also not be made, yet it is frequenctly used in circuits The opamp characteristic is approached closely, but never realized In fact, we could argue that the utility of any opamp comes from the fact that the opamp cannot be made perfect, since we ensure in practice that external components determine the characteristic of the device. That is, the idealization serves a pedagogic and practical purpose. Similarly, a negative impedance is a dependent object, only defined where larger positive impedances exist, since the negative impedance would charge the power supply from no source: another impossibility However, the understanding of oscillators was greatly facilitated by models using negative impedance. Similarly, positive time increments can exist only where there are normal time delays; they can be constructed by designing some of the circuit to have less delay than the rest The theoretical utility remains however.

Fixed and passive components

Let us define a passive circuit recursive definition for a perfect diode; for any value of Z—see Fig 11; and for an inverter, assuming we have a voltage summing circuit—see Fig. 12.

We can use the equivalence in Fig 13, so that

out+in+f(out)+out+out=out

and

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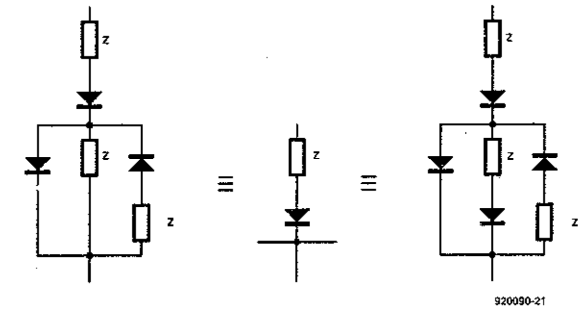

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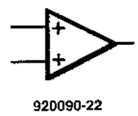
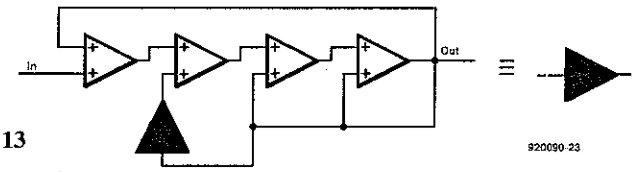


voltage summing circuit



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f(in) = out.

In+F(out)+2out=0,

so that

in+f^2(in)+2f(in)=0.

Let in=x and $f(x)=a_1x+a_2x^2+a_3x^3+$

$x(a_1y+a_2y^2+a_3y^3)+2(a_1x+a_2x^2+a_3x^3\ldots)=0,$

and

$y^2=a_1(a_1x+a_2x^2)+a_2(a_1x+a_2x^2)+a_3(a_1x+a_2x^2)+\ldots$

Equating the coefficients gives

$1+a_1^2+2a_i=0,$

which implies

$a_1=-1$; $a_i=0$; and i>1.

Thus, $f(x)=-x$, and we have recursively defined an inverter

Having defined an inverter, we can define a voltage-sum device (where output, $V=V_1+V_2$ at the inputs—see Fig 14).

That is,

f[f(out, in), out]=out,

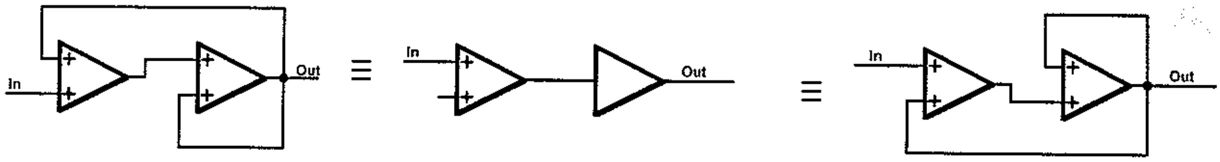
and

f(in, 0)=–out

Thus, $y=f[f(y, x), y]$

and $-f(x, 0)=y$.

From this, the rule f(in, 0)=–in can be deduced (see note at end)



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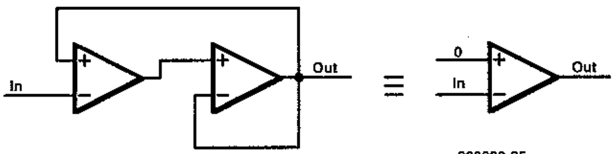
We may also define an opamp-like device: by a similar strategem:



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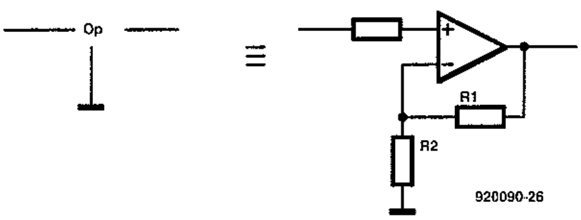
The usual opamp circuit:



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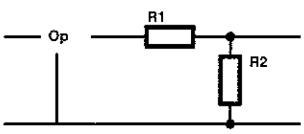
has a gain of $(R_1+R_2)/R_2$ Thus,



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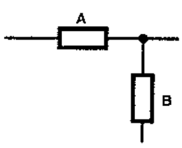
has a gain of I Therefore, we may describe the non-inverting mode of a *y* opamp as the inverse of a potential divider (which takes no current and is unloaded from a low impedance source)

In a repeated potential divider,



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let $r=B/(A+B)$; the new voltage ratio will be $Br^2/(B+Ar^2)$. When $A=B$, the sequence

$^1/_2$, $^1/_5$, $^1/_{13}$, $^1/_{34}$, ...

is obtained for this cascaded potential divider The sequence is the reciprocal of every other term of the Fibonacci series The matrix (un-augmented A-matrix)

$$\begin{bmatrix} 2 & R \\ 1/R & 1 \end{bmatrix} = M$$

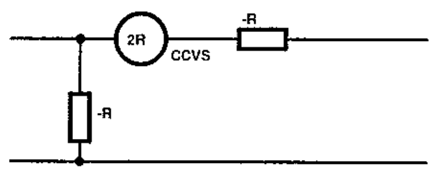
$$\begin{bmatrix} 5 & 3R \\ 3/R & 2 \end{bmatrix} = M^2$$

$$\begin{bmatrix} 13 & 8R \\ 8/R & 5 \end{bmatrix} = M^3$$

represents the situation. Note that the 0th term of the series is

$$\begin{bmatrix} 1 & R \\ 1/R & 0 \end{bmatrix}$$

which is a matrix interesting in any case It has the equivalent circuit



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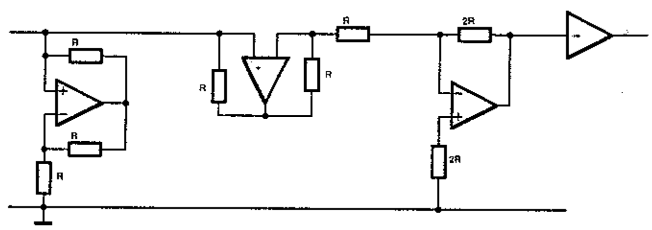
The practical circuit would be



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However, here we are concerned mainly with iterative schemes:

$$M_0 = \begin{bmatrix} 1 & R \\ 1/R & 0 \end{bmatrix}$$

Any voltage divider can then be written

$$\begin{bmatrix} 1 & \pm xZ \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & Z \\ 1/Z & 0 \end{bmatrix}^2$$

But consider first the iterated voltage divider with equal arms; let SET be the set we have:

$M_0 \varepsilon$ SET,

and if

$M \varepsilon$ SET, $MM_0 \varepsilon$ SET also

Let

$M_{-1}=M_0^{-1}$.

We then have the iterative scheme:

1. M_0 is a potential divider of this sort;
2. $M_{-1}MM_0=M$

Thus, we can characterize potential dividers by the extended scheme:

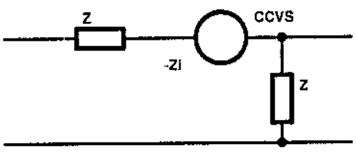
$$\begin{bmatrix}1 & xZ\\0 & 1\end{bmatrix}=X_x,\ \text{where } x \text{ is real}$$

1. $M_0 \varepsilon$ SET;
2. $M\varepsilon$ SET ⇨ $X_xM\varepsilon$ SET;
3. $M_{-1}MM_0=M$;
4. These are all, the smallest set defines them.

Hence, we have recursively defined a set of potential dividers. Some types of active circuit can be defined as the inverse of this class.

Let $X_xM_0^n=A$ be a potential divider; the A^{-1} is a non-inverting opamp of a certain type. Thus, we can also recursively define opamp circuits:

M_0^{-1} is



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whose matrix is:

$$\begin{bmatrix}0 & Z\\1/Z & -1\end{bmatrix}$$

Thus, A^{-1} is $M_0^{-n}X_x^{-1}$ and

$$\begin{bmatrix}1 & xZ\\0 & 1\end{bmatrix}^{-1}=\begin{bmatrix}1 & -xZ\\0 & 1\end{bmatrix}$$

$$M_0^{-2n}=\begin{bmatrix}F_{n+1} & -F_nZ\\-F_n/Z & F_{n-1}\end{bmatrix}\ \text{where } F_n \text{ are the Fibonacci series.}$$

$$M_0^{-2n}X_x^{-1}=\begin{bmatrix}F_{n+1} & -xZF_{n+1}-F_nZ\\-F_n/Z & F_{n-1}+xF_n\end{bmatrix}$$

Let us choose specific values for x and see what types of circuit emerge; the result may be constructive of a new approach. A negative value of x may be chosen to make a_{12} zero or a_{22} zero, so that we can choose either infinite mutual conductance or infinite current gain. If we choose infinite current gain, we are close to the opamp characteristic choosing Z negative for this non-inverting case. The method can also be used for other circuit elements, for example, the ideal full rectifier obeys these rules—see below and Fig. 23. Clearly, the

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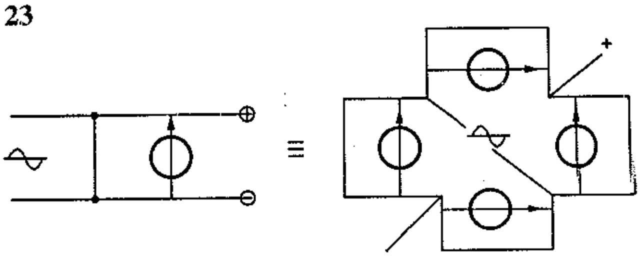
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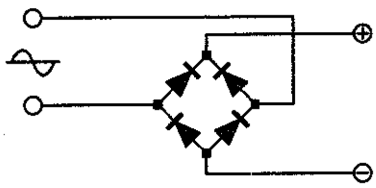
circuit is a form of idempotent

The question of whether these relations are definitive must be checked and, in fact, the definition of a single diode can be used to define these by the obvious method: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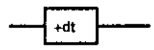
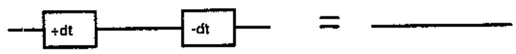
Here are some more facts—ptis again:

Let  denote a pti with positive time increment of dt, and  a normal time delay. Then,



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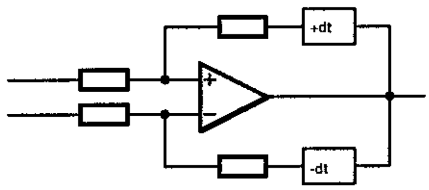
but a parallel connection is more fraught!



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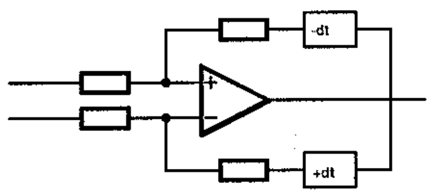
is unstable, whereas



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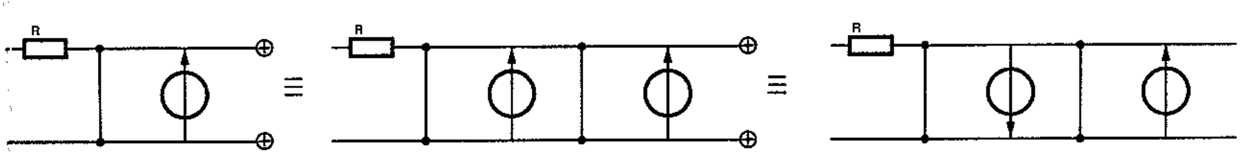
is, in some circumstances, not.



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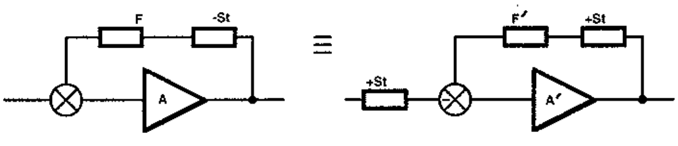
The simple relation



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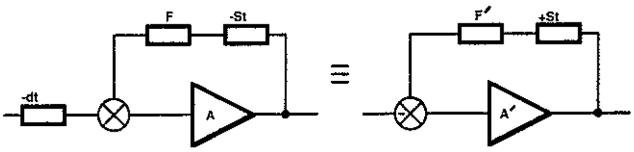
can be used to simulate the effect of a postive time increment element in feedback, since

$A'=g^{-1}$ and $F'=A^{-1}$



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The effect is very baffling to understand for those waveforms that require an infinite number of Fourier components to synthesize them. but straightforward for single frequencies The advantage is that a single frequency emerges with changed phase and can, therefore, be used to form a building block for any other waveform ■

Note Consider the conditions

$y=f[f(y, x), y]$ and $f(x, 0)=-y$

From symmetry it can easily be shown that

$f(0, 0)=0$ and $f(x, y)=f(y, x)$

Next consider

$f(x, y)=x+y$

We then have $-y=x$ and $y=[(y+x)+y]$, which fit the two equations; thus, $f(x, y)=x+y$ is one answer

Now consider $f(x, y)=x+y+d(x, y)$ and let $d(x, y)=ax+ay+bxy$ approximately for x, y very small Then,

$d[x+y+d(x, y), y]=-x-y-d(x, y)$

produces the result that $a=-1$, so that $-(cxy) -y +c(cxy^2)=-cxy$, which is impossible because x, y, though small, can vary independently of xy. The solution to this is that $d(x, y)=0$.
Thus, $f(x, y)=x+y$

***References*:**

Control Theory, Schaum Outline Series, McGraw-Hill.

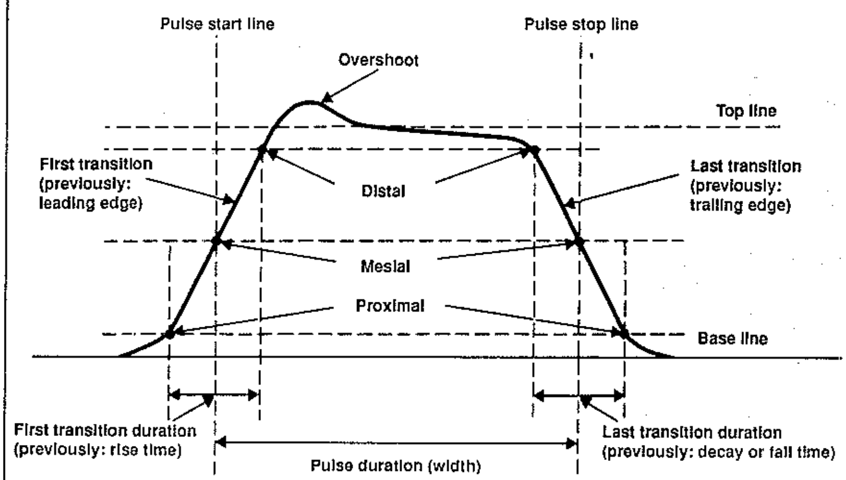
Cybernetics, Norbert Weiner, Di Stefano, McGraw-Hill

Fourier Analysis, Murray R Spiegel, Schaum Outline Series, McGraw-Hill.

Tables of Functions, Emde, Dover

Introductory Circuit Theory, Guillemin, Wiley.

Anatomy of a practical pulse



Anatomy of a practical pulse

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When it comes to describing a 'simple' pulse, or properties thereof, technical literature is sprinkled with vague, misleading and ambiguous terms and definitions What, for instance, does 'positive edge' mean when applied to a negative pulse? Is it the 'positive-going' edge, that is, in this case, the last transition, often called the 'trailing edge', or is it the first transition, often called 'leading edge'?

Why this confusion of terms and definitions has arisen is not clear. Both the British Standards Institution and the International Electrotechnical Commission have laid down agreed international terms and definitions, which are incorporated in the adjacent drawing. Further details may be obtained from British Standard BS 5698:1989 or IEC Standard IEC 469-1:1987. This magazine will continue using the standard terms and definitions applying to a pulse, although for a period the colloquial terms will be added in brackets where deemed necessary

Note also that the term 'duty cycle' is not used in connection with pulses; the correct term for the ratio of the pulse duration (width) to the pulse repetition period (pulse spacing) of a periodic pulse train is 'duty factor'.