SCIENCE & TECHNOLOGY

UNDERSTANDING WAVEFORM HARMONICS

by Dr K. A. Nigim

Many is the time when a strange unexpected or distorted waveform appears on your oscilloscope screen A lot of head scratching and book searching s usually needed to solve the mystery. However, these days, with the rapid advances in Computer Aided Engineering (CAE) software only a few steps or drag and click with the mouse attached to the computer are required to give you the answer instantly

Among the easy electronic circuit analyser software available at moderate cost and not too complex is the Micro-cap III electronic circuit package (about £120 or $200 student version) The program is exceptionally easy to operate in entering the electrical/electronic circuit into the PC (AT with math coprocessor advised) Simulation is performed on the circuit with realistic results A feature included in the package is Fourier Analysis, which evaluates the discrete Fourier transform of many distorted waveforms

It is not the aim of this article to focus on the software but rather use its programming facility to demonstrate and simplify the theory behind waveform harmonic content

Background

Fourier analysis is the mathematical ground for analysing the periodic or repetitive (and perhaps distorted) waveforms. Fourier theory simply breaks the waveform into several ideal sinusoidal waves that each has its own period and amplitude

Consider the waveform shown in Fig 1a It is said to consist of the two ideal waves shown in Fig 1b The first large wave is called the fundamental component and has an amplitude of, say, 100% at a frequency of 50 Hz The second smaller, wave is called the third harmonic with an amplitude of 30% at a frequency of 150 Hz.

In general, any periodical or repetitive waveform is defined by:

repetitive wave = DC component (A_0) + fundamental component (F_1) + harmonics (E_3)

In Fig 1, the waveform is represented mathematically by:

$$e_t = A_0 + A_1 \sin\omega t + A_3 \sin 3\omega t$$

If $A_3 = {}^1/_3 A_1$, the distorted wave is said to have 30% third harmonic.

Mathematically, any periodical wave can be represented in the form:

$$Y(\omega t) = A_0 + A_1\sin(\omega t) + A_2\sin(2\omega t) + \ldots + A_n\sin(n\omega t) + B_1\cos(\omega t) + B_2\cos(2\omega t) + \ldots + B_n\cos(n\omega t),$$

where

A_0 is the direct (constant) component;
A_1 A_n is the fundamental and odd harmonic components;
B_1 B_n is the fundamental and even harmonic components

Odd components exist when the wave is shaped by identical positive and negative cycles, that is, symmetrical around its axes Even components occur when the wave is shaped by non-symmetrical half cycles

Many CAE software packages present the wave by its vector form, which is then plotted in two informative scales. The first scale gives the magnitude and phase angle of the discrete harmonic components shaping the waveform. The second scale gives the cosine and sine values of the discrete harmonics. Both scales are related and either scale will be sufficient to determine the extent of distortion Mathematically, these scales are represented as follows

$$Y(t) = A_n \sin(n\omega t) + B_n \cos(n\omega t) = Y_n \sin(n\omega t + \Phi_n)$$

$$Y_n = \sqrt{A_n^2 + B_n^2} \Rightarrow \text{magnitude}$$

$$\Phi_n = \arctan\left(\frac{B_n}{A_n}\right) \Rightarrow \text{phase}$$

The terms A_n and B_n can be found from

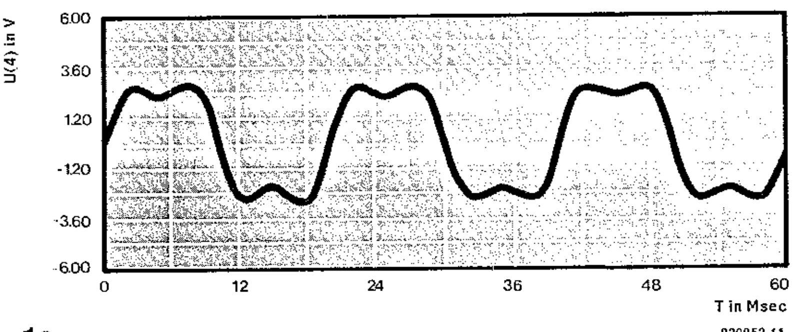
$$A_n = \frac{1}{\pi}\int_0^{2\pi} Y(\omega t)\sin(n\omega t)\,d(\omega t)$$

$$B_n = \frac{1}{\pi}\int_0^{2\pi} Y(\omega t)\cos(n\omega t)\,d(\omega t)$$

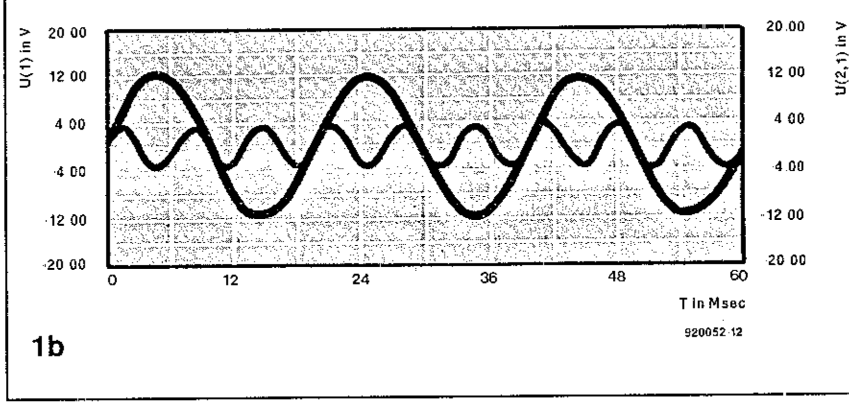
The DC component, if present is given by:

$$A_0 = \frac{1}{2\pi}\int_0^{2\pi} Y(\omega t)\,d(\omega t)$$

But wait. Do we really need to go into inte-



1a



1b

gration and vector calculations to understand waveform distortion? Relax, not in the 90s By using software available on the market and, of course, basic practice in electronic circuit analysis, it is quite possible to analyse almost any waveform and in effect produce the proper optimally designed system and this could be by introducing filters or by modifying the control concept

Sinusoidal waves

In this section, several AC shaped waves produced by a rectifier or phase-controlled power device are presented and their harmonic content is simplified

The ideal or smooth waveform is the sinusoidal wave that is produced by large generators in power stations Figure 2 shows its waveform and its Fourier analysis is presented in the shape of the harmonic magnitude and the phase angle The cosine and sine terms discussed in the previous section are plotted at the right. The analysis shown is the screen printout of the Micro-cap III software Fourier analysis section

It can be seen from Fig. 2 that the wave contains no DC nor any sort of harmonic, either odd or even. Thus the fundamental component dominates the wave shape.

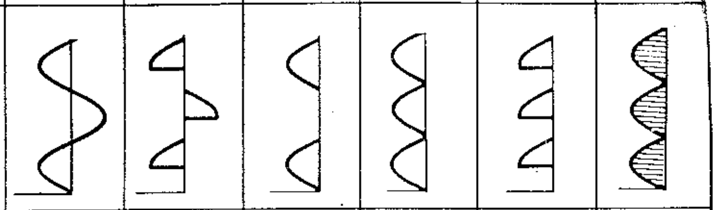
The absence of distortion means that no filtering is required and that all the generated energy is effectively transferred to its destination Although such pure waveforms exist along the electrical power lines they may not be so pure any more by the time they reach domestic or industrial power outlets because the mass of electrical and electronic equipment connected to these outlets nowadays generates a myriad of spurious frequencies that are transferred to the power lines One apparatus that is very sensitive to impure mains power is the computer which is why a power conditioner or UPS is normally recommended for its protection

Selected practical waveforms are compared with the ideal wave in Table 1

Wave I lists the ideal wave with its clear position of transmitting faithfully 100% power to the load without any distortion

Wave II shows a phase-controlled AC source as found in many light-dimmer circuits using bi-directional semiconductor devices to produce the desired variable AC source as the one shown For a 50% control ratio, that is, half the power transmitted, the print screen of the Fourier analysis shown in Fig 3 reveals a handful of harmonics despite the fact that only half the power is transmitted. Practically the level of harmonics can be neglected as long as its magnitude does not exceed 20% of that of the fundamental component

It is important to realize what this distortion might cause. High-frequency, high-level harmonics cause excessive heat loss and disturbance in the performance of the controlled system. All industrial and domestic systems are designed to operate from the ideal supply wave shape and frequency If a mains supply with a high content of harmonics is used to



	I	II	III	IV	V	VI
dc	0	0	63.5	150	70	150
Fund.	100	100	100	100	100	100
2	–	–	42	20	37	18
3	–	54	–	8	23	–
4	–	–	8	–	17	27
5	–	18	–	–	14	90
6	–	–	–	–	18	27
7	–	18	–	–	9	–
8	–	–	–	–	–	–
9	–	10	–	–	–	23
10	–	11	–	–	–	63

% of fundamental component

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Table 1. Fourier analysis of 'sinusoidal' waves.

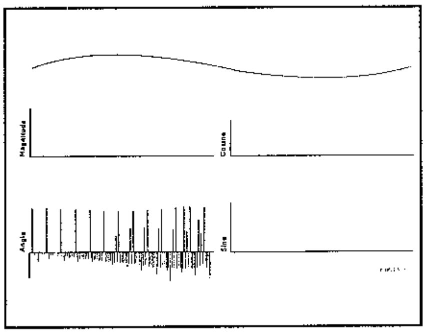


Fig. 2. Sinusoidal wave (ideal shape).

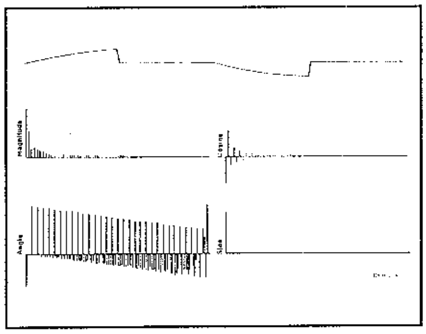


Fig. 3. Phase-controlled AC wave.

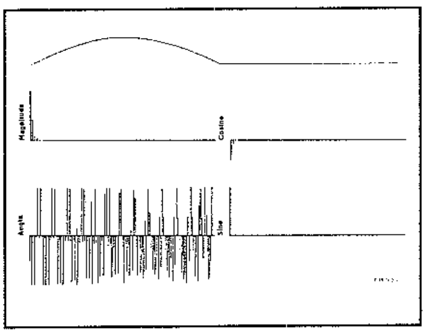


Fig. 4. Half-wave rectified wave.

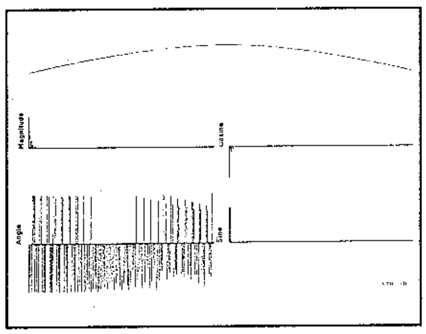


Fig 5. Full-wave rectified wave.

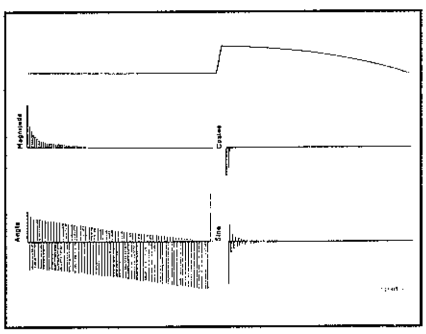


Fig. 6. Controlled (full) rectified wave.

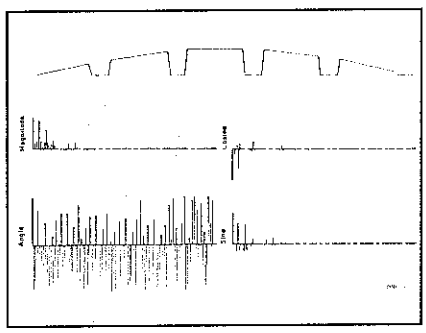


Fig. 7. PWM rectified wave.

power say, a universal motor such as hand drill or food mixer. the motor is faced with several AC supply signals at high frequencies and at different phase angles This causes a good deal of extra heat in the motor windings, noise and a drop in efficiency

More high-level harmonics contained in the wave cause excessive radio interference and requires wide-band filters Moreover, when a complex power source is supplied across a circuit containing inductance and capacitance, it may happen that the circuit resonates at one of the harmonic frequencies which is called selective resonance It is essential, therefore to include chokes or more complex filters or change the power source, both of which add to the final cost of the product

Wave III is a half-wave rectified wave that has a 61% DC level (amplitude/π) and a handful of odd harmonics Simply connecting a capacitor across the load is normally enough to reduce the harmonics to an acceptable level The Fourier analysis is plotted in Fig. 4

Wave IV is a full-wave rectified wave Its DC level is 150% of the fundamental, that is twice amplitude/π and, again a handful of odd harmonics that are, however, of a lower level than those in the half-wave rectified wave Again, a capacitor across the load will reduce the harmonic level substantially The Fourier analysis is plotted in Fig 5

Wave V is a full-wave rectified wave under phase control, resulting practically in a variable DC source. At a 50% control ratio, the DC level is 67% and the harmonic content is worse than in Wave III The Fourier analysis is plotted in Fig 6 Although this yields a variable DC source there is a problem in the filter selection: the smoothing capacitor chosen must be capable of by-passing high frequencies up to five times the supply frequency.

Wave VI is produced by pulse-width modulation (PWM) techniques to generate a variable DC source Depending on the rate of on/off switching, the harmonic content rises alarmingly, indicating the importance of well-designed filter elements for a wide range of frequencies. Te Fourier analysis is plotted in Fig 7 As the switching frequency increases, the harmonic level shifts to the higher frequencies which makes it difficult to attenuate them by simple filters.

From these examples. it is seen that distortion is at its worst when the supply changes state suddenly as in Waves III and VI. A smooth transition and composition of the wave has the least distortion and this makes filtering straightforward

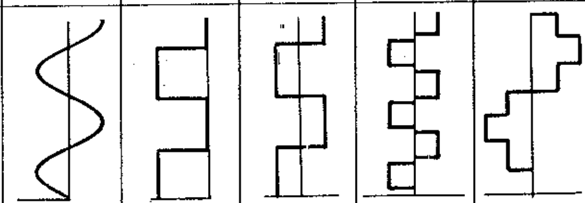
% of fundamental component	VII	VIII	IX	X	XI
					
dc	0	79	0	0	0
Fund.	100	100	100	100	100
2	–	–	–	–	–
3	–	33	33	–	–
4	–	–	–	–	–
5	–	20	20	18	20
6	–	–	–	–	–
7	–	14	14	15	14
8	–	–	–	–	–
9	–	11	11	–	–
10	–	–	–	–	–

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Table 2. Fourier analysis of DC or pulsed waveforms.

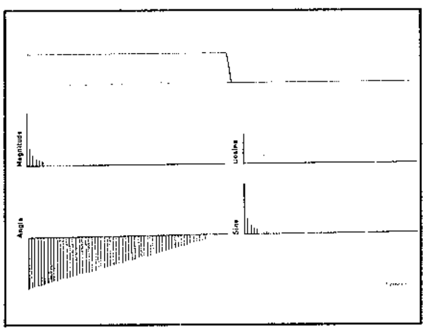

Fig. 8. Fourier plot of square wave.

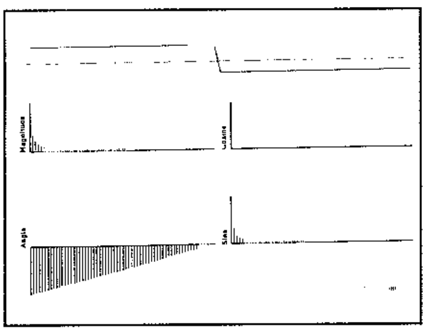

Fig. 9. Fourier plot of periodical DC wave.

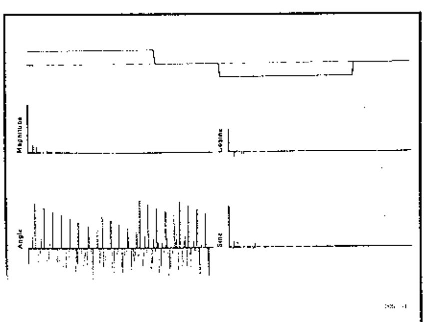

Fig. 10. Quasi-square periodical wave.

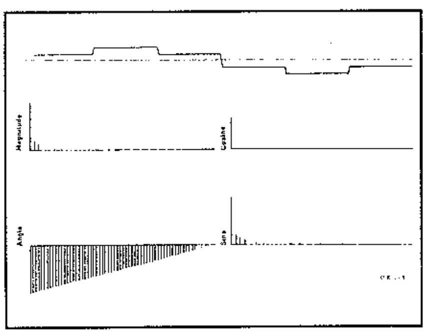

Fig. 11. Quasi-sinusoidal wave.

DC or pulsed waveforms

In this section, we look at harmonics in DC or pulsed waveforms—see Table 2

Wave VII is, again, the ideal sine wave

Wave VIII is a square wave as produced by for instance, a multivibrator oscillator By inspection the average value is half the amplitude for a 50% mark-space ratio. The fundamental is equal to twice the amplitude divided by π Only odd harmonics (all cosine terms are zero) are present owing to the nature of the wave. A low-pass filter with a transfer function equal to zero at high frequencies will by-pass high-order harmonics but will introduce phase distortion in the output Fourier analysis is plotted in Fig 8.

Wave IX is a periodical DC wave, typical of that generated by many commercial transistorized inverters An inverter is a circuit to convert a DC voltage into a periodical waveform The Fourier analysis is plotted in Fig 9 High-level third, fifth and seventh harmonics contribute to the shape of the wave Note that the more higher harmonics are contained, the sharper the wave will be

It is difficult to filter out the third and fifth harmonics as both are at high level and close to the fundamental. The filter design must be a good compromise between inductance and capacitance values Large inductances and small capacitances cause the inverter regulation to become poor Small inductances and large capacitances improve regulation but increase the current through the inverter. Regulation is the ratio between no-load and full-load output voltage

Wave X is, again, the inverter waveshape but with a different control Its Fourier analysis is plotted in Fig 10. The waveform, which has fewer harmonic peaks, is known as a quasi-square-wave. The analysis shows the existence of a small percentage of third harmonic content but the fifth and seventh are not noticeable

Wave XI is a quasi-sinusoidal wave that can be detected in many industrial motor drive controller The distortion over a wide frequency spectrum contains 20% fifth and 13% seventh harmonics, but at low level. Single filter elements are, therefore, effective in many cases If the smoothing filter in the DC path is capacitive, the system is called a constant-voltage source; if it is inductive, the system is a constant-current source. The Fourier analysis is plotted in Fig 11.

Summary

Most pulse-shaped waves contain odd harmonics If the periodical amplitude swings between equal positive and negative peaks, the average DC component is nought If the amplitude remains above the zero level, there is always a DC component.

The waveforms shown in this article are unfortunately all real-life shapes used intensively in the motion control of electric machines. Simple filters have not much effect, and are often bulky and expensive The existence of harmonics produces torque pulsation in motors, which causes extra heat vibration and noise

With the advances of microprocessor-controlled inverters, harmonic attenuation by pulse-width modulation is now available As always however one should consider complexity and cost against performance ■

RED-LIGHT DIODE LASERS

based on an original article by S. von Fehren

DIODE lasers that operate from the near-ultraviolet to well into the infra-red region are commonplace and used, among others in optical fibre communication, optical memories and compact-disc players Until a few years ago, the only lasers producing visible light were He-Ne lasers. Some of the He-Ne lasers emit a deep red light at a wavelength of 632.8 nm, that is in the visible light region, while others operate at 1.15 µm or 3.39 µm In spite of the fact that they are (relatively) large, require a power source of kilovolts, cannot be modulated and are highly sensitive to mechanical phenomena, these gas lasers have become very popular because they are inexpensive and easy to manage.

These lasers now have a serious competitor in a diode laser, announced by Toshiba in 1987, but not commercially available until recently that operates at 660 nm (red light).

Diodes that emit visible light have of course, been available for many years Such light-emitting diodes (LEDs) are made from a variety of semiconductor material to obtain a particular colour of light. Gallium-arsenide (GaAs) for near-infra-red; gallium-arsenide-phosphide (GaAsP) for red and yellow; gallium-phosphide (GaP) for green and blue See Table 1 for light and near-light wavelengths and associated colours

Without repeating its derivation (see Ref 1), the formula from which the wavelength λ can be calculated is

$$\Delta E = h\nu = hc/\lambda, \qquad [1]$$

where ΔE is the energy released when an electron passes from one energy level to another, h is the Planck constant (4.14×10^{-15} eV s), ν is the frequency of radiation, c is the speed of light in a vacuum and λ is the wavelength (colour) of the emitted light The quantity $h\nu$ is a quantum of energy commonly called a photon

From formula [1], it is seen that since h and c are constants, the wavelength depends entirely on ΔE. In semiconductors, energy difference is called the energy gap, expressed in eV (electronvolts). One eV = 1.60210^{-19} joule (J) The energy gap cannot be measured: it is determined empirically, that is, by measurement The energy gap between gallium and arsenide is 1.4 eV. Entering the various quantities in formula [1] results in a wavelength:

$$\lambda = hc/\Delta E =$$
$$= 4.14\times10^{-15}\times 3\times10^{8}/1.4 =$$
$$= 8.86\times10^{-7}\ \text{m} = 886\ \text{nm},$$

which is in the near-infra-red region.

To obtain shorter wavelengths (that fall in the visible region), different materials must be used

Standard laser diodes are made from gallium-aluminium-arsenide (GaAlAs), which has an energy gap of 1.6 eV corresponding to a wavelength (computation as before) of 775 nm. Because of high production quantities and standardized manufacture, these diodes cost only about a few pounds ex-factory They are used extensively in CD players and laser printers

New techniques

The new laser diodes, Toshiba Types TOLD9220 and TOLD9222, are made from indium-gallium-aluminium-phosphide (InGaAlP) which has an energy gap of about 1.9 eV to give a wavelength of 660 nm Prices of these devices have been coming down rapidly

Currently, Toshiba research scientists are working on a laser diode that will emit blue light (500–445 nm) The difficulty, as before, is finding suitable semiconductor materials that can be doped appropriately to yield a p-n junction with the required energy gap of about 2.2 eV.

The (index-based) construction of the TOLD9220 is shown in Fig 1 Its major parameters (at 25 °C) are

- wavelength, λ: 660 nm
- threshold current I_{th}: 75 mA (max 90 mA)

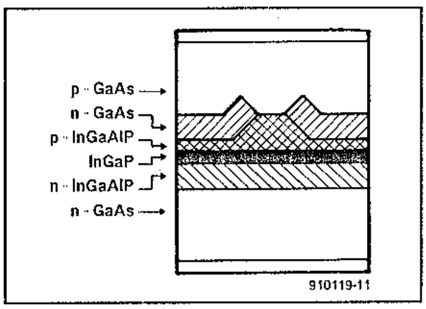

Fig. 1. Layer construction of the new Toshiba laser diodes

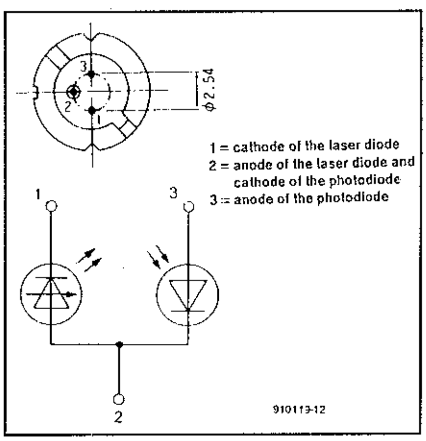

Fig. 2. Pinout of the TOLD9220.