FOUR-TERMINAL NETWORKS - PART 2

The design of attenuators

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In this second and final part of our investigation into the design and uses of four-terminal attenuator systems, we shall first of all derive some simple relationships between the characteristic resistance of an attenuator T-section and the degree of attenuation provided by the section in terms of the resistive elements that make up the section.

THE calculations are not difficult, and those readers with an arithmetical bent might like to confirm some of the answers We will relate our results with those of the π–section, consider insertion loss, and have a look at the practical design of attenuators

Finding values

For the properly terminated symmetrical T-section shown in Fig 8, we let the attenuation be expressed as $N=U_1/U_2$, in which numbered subscripts are used for convenience Since the input resistance is R_o, the input current, I_1, must be U_1/R_o and the voltage, U, across the centre shunt arm will be

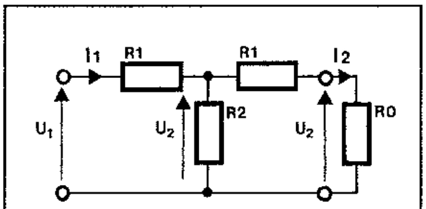

Fig. 8. Deriving the attenuation factor of a section.

$$U=U_1-I_1R_1$$

Hence,

$$U=U_1-U_1R_1/R_o=U_1[1-R_1/R_o]$$

But the output voltage, U_2, is clearly the proportion of U developed across R_o, given by

$$U_2=U[R_o/(R_1+R_o)],$$

whence

$$U_2=U_1[1-(R_1/R_o)][R_o/(R_1+R_o)]$$

From this, the ratio

$$U_1/U_2=N=(R_o+R_1)/(R_o-R_1)$$

This gives the attenuation factor in terms of R_o and R_1. What we need now is a relationship involving R_1 and R_2 in terms of R_o and N Working from this last result, we find that

$$R_1=R_0[(N-1)/(N+1)].$$

Using the earlier result that

$$R_o=\sqrt{(R_1^2+2R_1R_2)},$$

we get

$$R_2=R_0[2N/(N^2-1)]$$

When R_o and N are known, these simple expressions enable an attenuator to be designed that gives a desired signal reduction and the proper matching conditions for the circuit system concerned. Figure 9 illustrates the meaning of the two formulae

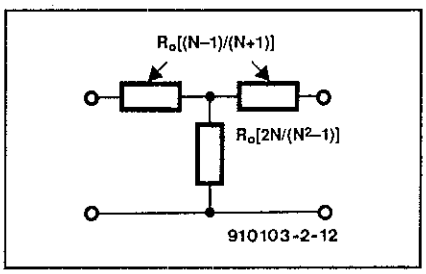

Fig. 9. Relationships to determine the elements in terms of R_o and N

Trying things out

A couple of examples will show how we can apply these results to solve some elementary design problems.

First, suppose we want a symmetrical T–section network to match into a 300 Ω line and have a voltage attenuation of 14 dB; what values of resistors do we want?

Well, the attenuation factor N=antilog 14/20 =antilog 0 7=5 In other words, the output will be one fifth of the input We thus get

$$R_1=300[(5-1)/(5+1)]=300\times 4/6=200\ \Omega;$$

and

$$R_2=300[(2\times 5)/(25-1)]=300\times 10/24=125\ \Omega$$

Figure 10 shows the completed section

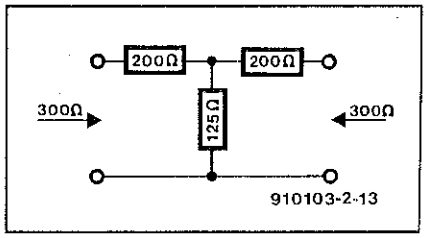

Fig. 10. Example of en elementary T-section design.

Suppose now that the output stage of a small transmitter has an internal resistance of 600 Ω and is intended to supply current to a 600 Ω load We need to design a T-section which, when inserted into the line connecting generator and load, will reduce the load current to one third of its initial value

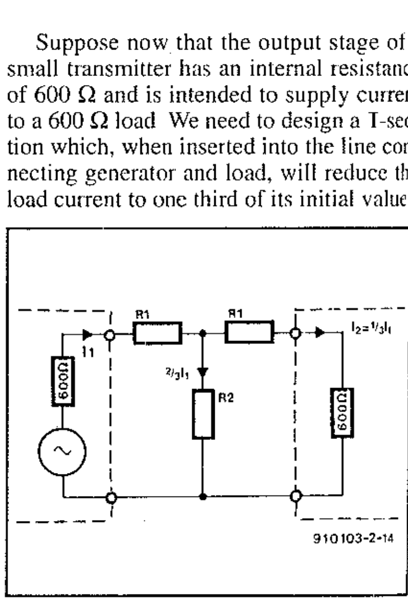

Fig. 11. A further design example relating to correct matching procedures.

This situation can be illustrated after the manner of Fig. 11 The section matches between equal impedances of 600 Ω and should therefore, have R_o=600 Ω also The attenuation factor, N, is 3 so that

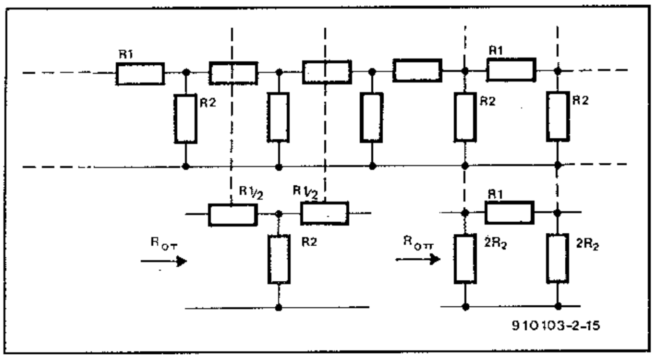

Fig. 12. These attenuator sections have the same total series and shunt resistances; then, $Z_{0T}Z_{0\pi}=R_1R_2$

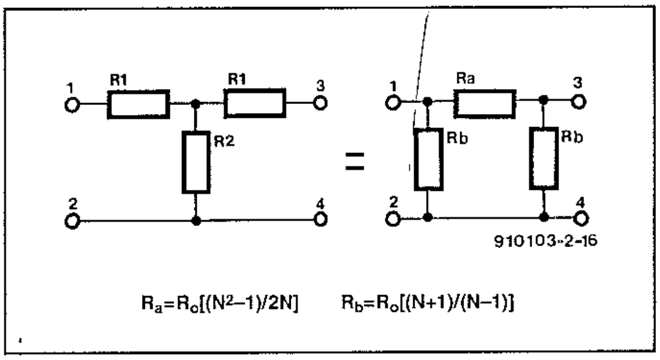

Fig. 13. Finding the element relationships between the T- and π–sections.

$R_1=600[(3-1)/(3+1)]=600\times{}^2/_4=300\ \Omega$

and

$R_2=600[(2\times3)/(9-1)]=600\times{}^6/_8=450\ \Omega$

In a case like this, it is important to notice that current I_1 supplied by the generator is the *same* whether the section is in circuit or not. In both cases, the generator sees a resistive load of 600 Ω. With the section connected, the 'unwanted' current $({}^2/_3 I_1)$ flows along the shunt arm, as an application of Ohm's law will immediately verify.

The π-section

It might appear that the π-section has been rather neglected but there is a circuit relationship between the T-section and π-sections that makes an analysis of the π-section quite easy. Figure 12 shows a ladder network of series and shunt resistances; from an examination of this network, is it a chain of T-sections or π-sections? The answer is that it depends how you look at it. For either a T-section can be taken from the network by considering the line division AA, or a π–section taken by considering the line division BB. The T-section cuts through the centre of each of the series elements R_1, while the π-section effectually splits the shunt element into two parallel parts, each of value $2R_2$. Hence the ladder can be viewed as a string of T-sections or π-sections.

We might expect, therefore, that there would be little difference between the relationships derived for the two types of section, but for the matter of *equivalence* it isn't just a job of halving the series arms and doubling up the shunts as this last analysis might have led us to believe.

We can make an exact comparison between the sections by looking at Fig. 13. Here, we use letter subscripts for the π-section resistors to avoid confusion.

For the two networks to be equivalent in their characteristics, the resistance seen between terminals 1 and 2 of both networks must be equal, as must the resistance between terminals 1 and 3 of both networks, with the output on open circuit. This means that R_1+R_2 on the T-section must equal $R_b(R_a+R_b)/(R_a+2R_b)$ on the π-section. Also,

$$2R_1=2R_aR_b/(R_a+2R_b)$$

or

$$R_1=R_aR_b/(R_a+2R_b)$$

Substituting back for R_1 in the first expression then gives us

$$R_2=R_b^2/(R_a+2R_b)$$

These relationships provide us with the elements of the T-network such that this will correspond to a given π-network.

It is also not too hard to demonstrate that for the basic π-section shown in Fig. 13

$$R_0=R_aR_b/\sqrt{(R_a^2+2R_aR_b)}=\sqrt{R_{oc}R_{sc}}$$

and that

$$R_a=R_0[(N^2-1)/2N]$$

and

$$R_b=R_0[(N+1)/(N-1)]$$

where the bracketed term is simply inverted from that of the T-section and R_a and R_b are themselves swapped over.

A useful relationship follows from the first of these: since $\sqrt{(R_1^2+2R_1R_2)}$ is the R_0 of a T-section (R_{0T}), we can deduce that for sections having the *same total series and shunt resistances* (see Fig. 12 again)

$$R_{0\pi}R_{0T}=R_1R_2=R_aR_b$$

Don't confuse this with the equivalence relationships.

So, fundamentally there is no difference in the functions of these alternative forms of the sections and either may be used in a particular situation. The actual choice depends upon which form, given a particular value of characteristic resistance R_0, yields the more readily obtainable values of resistance for the elements.

In general design work, where stringent conditions are not of vital importance, the use of 5% resistors is quite acceptable; the resulting variation in attenuation will normally be no more than 0.5 dB and the mismatch in the characteristic resistance itself of the order of 5%. We will return to these points a little later on.

Insertion loss

The important thing in designing attenuators for general bench use is not to bother too much about getting exact answers to the calculations. It is no use working out a resistance to three or four decimal places and

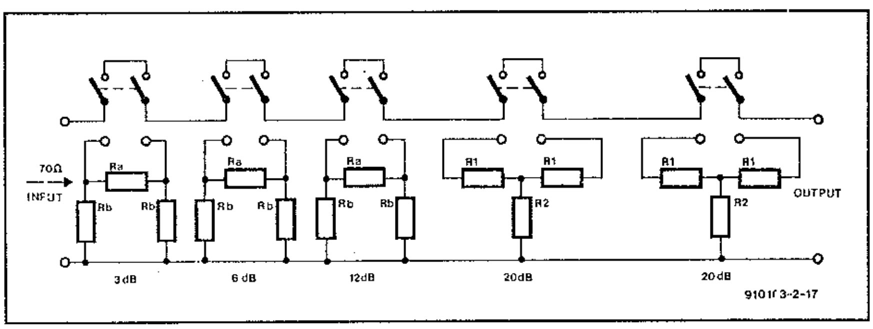

Fig. 14. Designing an attenuator knowing R_0 and the required attenuation.

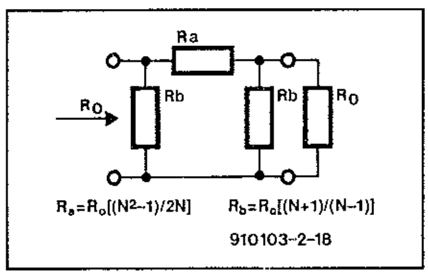

Fig. 15. The relationships should be compared with those for the T-section given earlier.

then discover that you've got to use a preferred value anyway The attenuation factor can often be rounded off in the same way; 6 dB, for instance is a voltage ratio of 1 9953. but we wouldn't put this into a formula: we use the nice round figure of 2 Of course things don't always work out quite so conveniently but anything beyond two decimal places is a waste of effort

Let us now design a 5-section attenuator for a 70-Ω line giving switched positions of 3 dB, 6 dB 12dB and two 20 dB reductions For interest, let the first three stages be derived from π-sections and the last two from T-sections The job will look like Fig 14, which also shows the appropriate switching

Look first at the 3-dB section; the required attenuation is 3 dB for which attenuation factor N is found to be antilog $^3/_{20}$ or 1 41 Using the relationships shown for convenience in Fig 15. and with R_0=70 Ω we find that R_a=24.8 Ω and R_b=410 Ω The last value is a bit of a problem since it falls between 390 Ω and 430 Ω in the E24 range One way out is to parallel 9 1 kΩ with a 430 Ω or get hold of a 412 Ω from the E96 precision range Otherwise, use 24 Ω and 390 Ω in series; the mismatch is not very serious: R_0 then works out at 66 Ω

Going through the same procedure for the 6 dB (N=2) and the 12 dB (N=4) π–sections we find for the elements the respective values: R_a=52 Ω; R_b=210 Ω; R_a=131 Ω; R_b=116 Ω Practical values here would be 51 Ω; 220 Ω; 130 Ω; and 120 Ω Amuse yourself by calculating the above three ranges in terms of T-sections; do the resistance values fit any better?

For the two T-sections we require an attenuation of 20 dB (N=10): hence, for R_0=70 Ω we find

$$R_1=R_0[(N-1)/(N+1)]=70\times{}^9/_{11}=57\ \Omega$$

and

$$R_2=R_0[2N/(N^2-1)]=70\times{}^{20}/_{99}=14\ \Omega$$

Practical values here would be 56 Ω and 13 Ω. The completed attenuator is shown in Fig. 16

Now try designing a series of five T-sections with switched ranges of 1 dB, 2 dB, 4 dB, 8 dB and 16 dB; this enables a maximum attenuation of 31 dB to be achieved in 1 dB steps The R_0 should suit your own particular fancy As a help, the respective N values are 1 12; 1 26; 1 58; 2 45 and 6 30

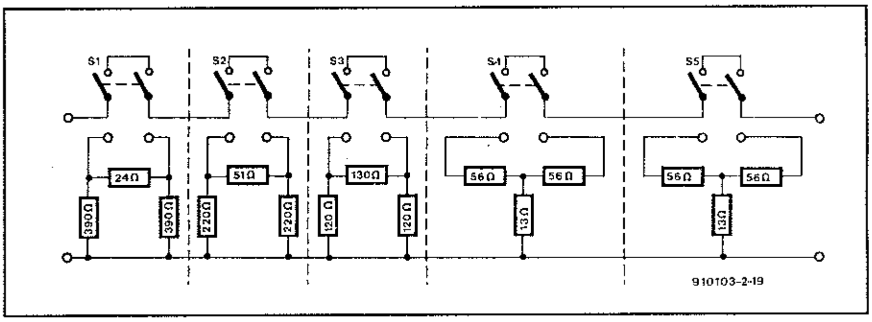

Fig. 16. The completed design.

Practical considerations

When we talk about purely resistive attenuators we are of course, in the realms of fantasy; it is not possible to make up an attenuator system having a number of sections in tandem without introducing some inductive and capacitive elements The object of any design is to keep these to their absolute minimum, just as in filters, where inductance and capacitance are the necessary elements, we try to eliminate resistance

It is necessary then, to keep all interconnecting leads as short as possible and to avoid the proximity of these leads as well as the resistances themselves, to any surrounding metal parts Further there must be no capacitive coupling between the sections or certain frequency components of the signal will sneak through without the desired attenuation, but with definite phase shifts The resistances should not for obvious reasons, be wirewound, though types with a non-inductive construction might be used.

Simple aluminium boxes are available nowadays that can be used to house bench-type attenuators Types measuring some 120–150 mm in length with perhaps 50 mm width and a depth of about the same are ideal Figure 17 shows the general method of assembling such attenuators

The box is divided up into the required number of compartments by cross screens that can be made of thin aluminium or tinplate If aluminium is used the screen will have to be flanged and screwed to the side walls of the box, but with an all tinplate construction soldering is the best approach

Small holes are drilled centrally in each screen (before fitting!) to permit the series-arm resistors to feed through, and the shunt-arm resistors are returned either directly to each screen or to a convenient position on the box floor itself A relatively heavy common wire running through the screens sometimes makes dependence on the box metal unnecessary

Slide or miniature toggle switches are used for the attenuation selection though it is possible with certain forms of construction—see Fig 18—to use rotary switches It is often easier to use the base of the box to carry the terminals and switching, the actual lid being fitted last and becoming the working base This avoids having long leads to connect the switches which would then be separated from the rest of the network in the body of the box

Attenuators that are to be housed inside equipment such as a signal generator for example can be built into the general internal design method but a divided-compartment assembly is still necessary if the best results are to be obtained For less stringent work the resistances can often be simply mounted directly to the tags on a rotary switch wafer or wafers as they could for the system shown in Fig. 18 Most simple 'dividers' are assembled this way.

The power ratings of the resistors used in attenuators must of course be such as to ensure that no appreciable temperature rise can occur in normal use ■

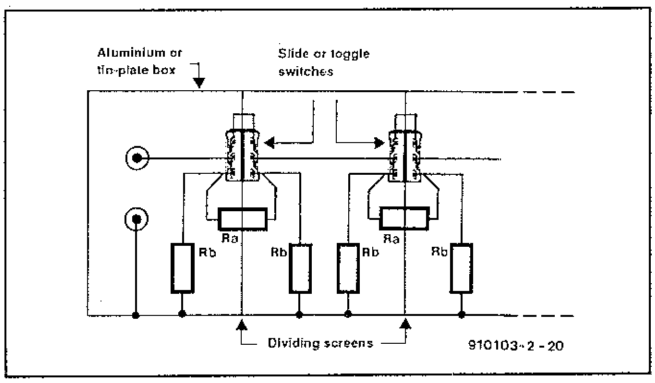

Fig. 17. Practical method of assembly.

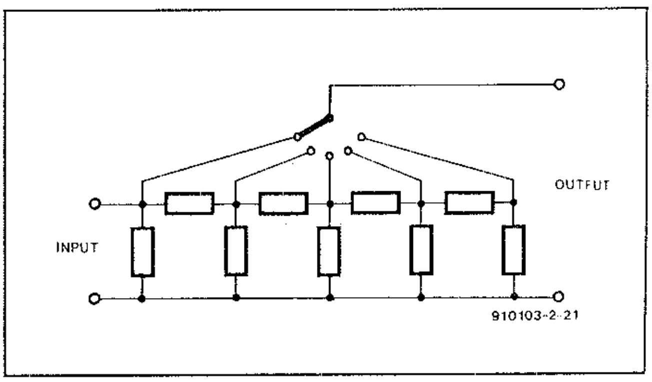

Fig. 18. An alternative switching system—more difficult to screen adequately.