RECTIFIER CALCULATIONS

What is the ripple voltage across the buffer capacitor? What is the level of the peak current through the rectifier diodes? The values of these and many other quantities are often obtained by rule of thumb, but what is the basis of that? This article aims to answer that question.

A TYPICAL, fundamental mains power supply is shown in Fig. 1. In this, U_p is the primary voltage; R_p represents the total losses at the primary side; U_s is the secondary voltage; R_s represents the total losses at the secondary side; U_C is the open-circuit output voltage (e.m.f.).

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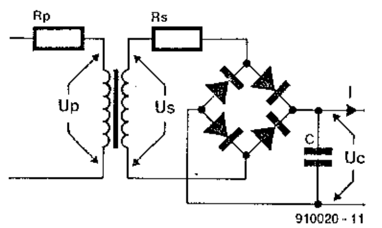

It is advantageous in most calculations to combine R_p and R_s into a single loss representation at the secondary side and call this R. The value of R (in Ω) is given by

$$R = (U_s/U_p)^2 R_p + R_s \qquad [1]$$

This means, in effect, that the transformer need no longer be a part of any calculation; the circuit is fed by a source that provides a voltage U_s has an internal resistance R and, for nominal loads, has negligible inductance.

Unfortunately, R can not be calculated readily owing to lack of data, and measuring it is also not possible. An ohmmeter will not do because that does not take into account the losses caused by stray magnetism. Similarly, the resistance offered by the diodes can not be measured realistically.

For a proper measurement to be made, a variac is needed at the primary side of the transformer (begin with $U_p = 0$). The buffer capacitor, C, is short-circuited by an ammeter (whence the variac).

Start by adjusting the variac until the secondary winding provides the nominal level of current specified by the manufacturer. The primary voltage needed for this depends on the variac, the transformer, the rectifier diode(s), and the ammeter. The current indicated by the ammeter is the r.m.s. value of the short-circuit current I_{sc}. Now R (in Ω) may be calculated from

$$R = U_s / I_{sc} \qquad [2]$$

Draw first, calculate later

Figure 2 shows a few voltages that are involved in the calculations. If rectifiers were ideal components the waveform of U_s would indeed be as drawn. Since, however, ideal components do not exist, the voltage that is available to charge the buffer capacitor is lower than U_s by the knee voltage of the diodes, U_d. The level of that knee voltage depends on the number of diodes and on the forward voltage of each diode. The bridge rectifier in Fig. 1 will lower U_s by about 2 V. The voltage, U, after the rectifiers is thus

$$U = U_s - U_d \qquad [3]$$

In most textbooks, the waveshape of that voltage is shown slightly differently: roughly as the dashed line in Fig. 2 immediately below the peak of U. In reality, the voltage will have a shape somewhere between the dashed and solid curves of U_C depending on the ratio charging current : discharge current.

The shape of the (solid) curve of U_C is explained as follows. From the moment that U becomes larger than U_C current flows from the voltage source to the buffer capacitor via resistor R. The level of the current is determined by the difference between U and U_C, which is U_R and the value of R across which that difference voltage exists.

At the onset, U_R is tiny and the charging current, I_{ch}, is smaller than the discharge current I_{dis}: U_C will then drop to U_{min} (the minimum value of U_C). From there U_C rises and may go on rising after U has begun to drop again (because I_{ch} is then still larger than I_{dis}). However, at a given instant U becomes too small and the capacitor starts to discharge. How much U_C will drop depends on the current, I_L, that the capacitor must supply to the load and on the duration, t_{dis}, of the discharge current.

The level of load current is known from the specification of the power supply. The duration of the discharge current is equal to the period of the rectified voltage minus the time necessary to charge the capacitor. The charging period is (as yet) unknown, but in general it is much smaller, in fact negligible, compared with the discharge period. That means that the discharge time is equal to the period, T_r, of the rectified voltage. Note that T_r depends on the frequency and the method of rectification. For instance, in full-wave rectification, the capacitor is charged twice as fast as in half-wave rectification. That means that in half-wave rectification $T_r = 1/f$ and in full-wave rectification $T_r = 1/2f$.

The charge, Q, that the capacitor can supply in that time is given by

$$Q = IT_r \qquad [4]$$

The ripple voltage, U_r, may be calculated from

$$U_r = Q/C = I/2fC \qquad [5]$$

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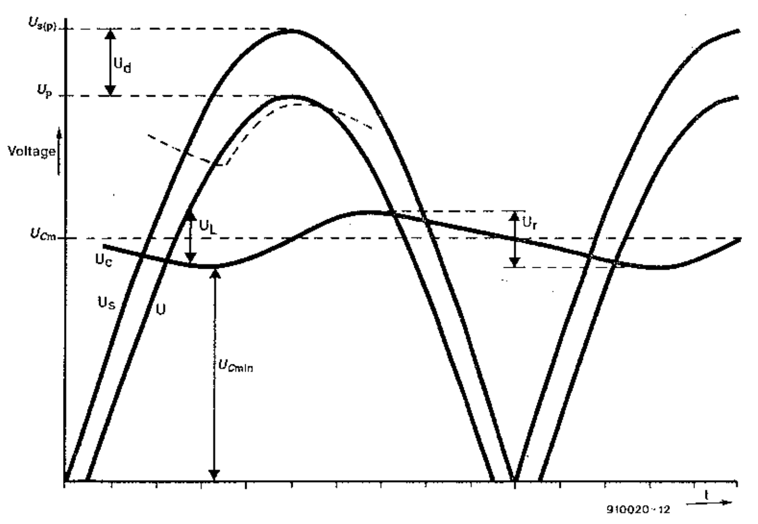

Apart from the ripple voltage the mean capacitor voltage U_{Cm} or, if voltage regulators are used the minimum capacitor voltage $U_{C\,min}$ is important The exact computation of these voltages is unfortunately fairly complex and therefore in this article approximations are used based on the data that have been found so far Which of the two approximations given must be used for the mean capacitor voltage depends primarily on the ratio $R:R_L$ where R represents the total resistance of the transformer and the rectifier and R_L is the load through which current I_L flows If R_L is large with respect to R (the usual case) I_L is small with respect to the maximum current with which the buffer capacitor can be charged This means that the capacitor can be charged to the peak value, U_p, of the available voltage The mean capacitor voltage is then

$$U_{Cm} = U_p - {}^1\!/_2 U_r \qquad [6]$$

and the minimum capacitor voltage is

$$U_{C\,min} = U_p - U_r \qquad [7]$$

If R_L is not much larger than R, the capacitor will not charge to U_p and [6] and [7] are no longer valid However U_{Cm} may then be calculated with the aid of Fig 3.

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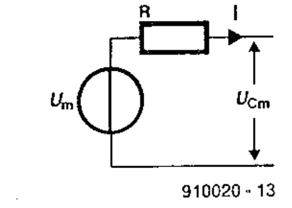


The voltage source, which represents the transformer and rectifier provides a direct voltage that is equal to the mean value of the non-smoothed direct e.m.f. U, supplied by the transformer and rectifier The internal resistance of the source is represented by R. If a current I is drawn from the circuit that is the circuit is loaded, the output voltage U_C will reduce by the voltage drop across R that is

$$U_C = U - IR \qquad [8]$$

The level of U is dependent on the method of rectification: with full-wave rectification it is equal to $2U_p/\pi$ and with half-wave rectification it is equal to U_p/π

The maximum current through the rectifier diode(s) $I_{d(p)}$ is

$$I_{d(p)} = U_p/R \qquad [9]$$

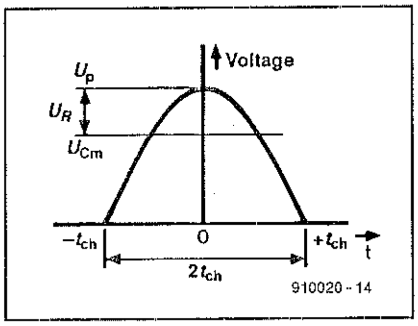
This level of current flows normally only when the power supply is switched on since the capacitor voltage is then zero. It can, however also flow in conditions of very heavy (over-) loads when the minimum of the ripple voltage is zero Normally however the capacitor voltage is appreciably higher and the current that then flows is equal to the maximum voltage drop across R (roughly $U - U_C$) divided by R that is

$$I_d = (U - U_C)/R \qquad [10]$$

The rectifier diodes must be able to cope continuously with this level of current, and should also be able to withstand for short periods the maximum current $I_{d(p)}$

The considerations in this article allow the practical design and calculation of the rectifier section of a power supply The formulas given are not one hundred per cent accurate for all theoretical considerations; for instance, no account has been taken of the time constant τ presented by resistance R and buffer capacitor C ■

Some additional mathematics



The buffer capacitor is charged via resistor R for a period $-t_L$ to $+t_L$ The charge, Q, stored in the capacitor in that period depends on the average charging current, I_{ch} and the length of the period, $t = 2t_{ch}$:

$$Q = I_{ch} t \qquad [11]$$

The average current depends on the resistance R, in the charging circuit and the average voltage, U_R, across that resistance:

$$Q = U_R 2t_{ch}/R \qquad [12]$$

So that

$$U_R 2t_{ch} = 2U_p \int_0^{t_{ch}} \cos(\omega t)\,dt =$$

$$= 2U_p \sin(\omega t_{ch}) \qquad [13]$$

and

$$Q = \frac{2U_p \sin t_{ch}}{\omega R} = \frac{2I_p \sin t_{ch}}{\omega} \qquad [14]$$

where I_p is the peak value of the current that the supply can deliver for short periods, that is, U_p/R

To keep the mean value of the capacitor voltage constant, the charge removed from the capacitor must equal the input charge, that is,

$$I_{ch} 2t_{ch} = I_{dis} T_r \qquad [15]$$

where I_{dis} is the discharge current and T_r is the period of the rectified voltage (as explained in the text, $T_r = 1/f$ in half-wave rectification and $= 1/2f$ in full-wave rectification).

Since the incoming and outgoing charges are equal,

$$\sin(\omega t_{ch}) = \omega I_{dis} T_r / 2I_p, \qquad [16]$$

so that,

$$t_{ch} = \frac{1}{\omega} \arcsin\left(\frac{\omega I_{dis} T_r}{2I_p}\right) \qquad [17]$$

Depending on the method of rectification, $\omega T_r = \pi$ (full-wave rectification) or $\omega T_r = 2\pi$ (half-wave rectification). If it is assumed that I_p is several times (or even many times) larger than I_{dis}, the following approximations are obtained:

$$t_{ch} = \frac{\pi I_{ch}}{2\,\omega I_p} = \frac{I_{ch}}{4fI_p} \qquad [18]$$

or

$$t_{ch} = \frac{2\,\pi I_{ch}}{2\,\omega I_p} = \frac{I_{ch}}{2fI_p} \qquad [19]$$

where it is assumed that

$$\arcsin(x) \approx x \text{ if } x \leq 0.5$$

The level of the ripple voltage, U_r, is determined by the amount of charge removed during the period that the capacitor is not being charged. The length of that period is $T_r - 2t_{ch}$, so that

$$U_r = Q/C = I_{dis}(T_r - 2t_{ch})/C \qquad [20]$$

In a well-designed power supply, the capacitor is charged rapidly and $t_{ch} \ll T_r$, so that a simplification may be made:

$$U_r = I_{dis} T_r / C \qquad [21]$$

or

$$U_r = I_{dis}/2fC \text{ (full-wave rectification)} \qquad [22]$$

or

$$U_r = I_{dis}/fC \text{ (half-wave rectification).} \qquad [23]$$