LINE PULSE FUNDAMENTALS:

SOME PROBLEMS UNKNOTTED

By Bryan Hart

Introduction

Throw a stone the size of a golf-ball into a can of water the size of a tea-cup and see what happens: virtually all points on the surface of the water are disturbed simultaneously. This is a rough mechanical analogy to the case of a 'lumped' electrical circuit e.g. a simple resistive potentiometer, comprising two resistors subjected to a transient input.

Throw the same stone into the middle of a village pond and observe a different effect: all points on the surface of the pond are not affected simultaneously: they are disturbed only as ripples move outward from the point where the stone falls. This is a crude mechanical analogy to the case of a 'distributed' electrical circuit, notably a transmission line, with a transient input.

The difference between the two cases arises through the finite time taken for disturbances to be transmitted. A variation of the pond analogy which has long been used in the study of wave transmission, is the 'canal' analogy. In this, we consider what happens when a straight plank is dropped into a canal in a direction perpendicular to its length. Straight ripples parallel to the length of the plank move outward from the place where it falls. This analogy is more appropriate in the discussion that follows, because propagation is characterized by movement principally in one dimension.

Transmission lines are very important in digital electronics because of their use in the distribution of fast logic signals but their operation is sometimes a puzzle to budding engineers (some with a predominantly mechanical engineering background) who have been taught the basic principles of lumped-circuit electronics but who have not studied established Electromagnetic Theory (or been convinced by it, even if they had!)

This introductory article sets out to clarify the understanding of some fundamental aspects of the pulse operation of transmission lines particularly the popular twisted pair line (t.p.l.). The aim is to concentrate on the basic circuit theory aspects and practically observable waveforms that support the theoretical background.

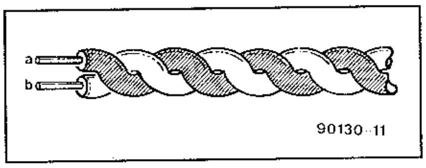

Fig 1 A twisted pair line (t.p.l.)

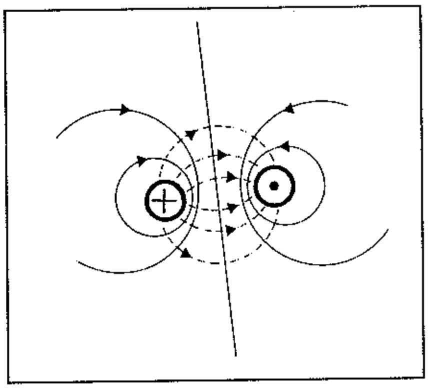

Fig 2 Field patterns at a point on a t.p.l. under d.c. conditions. Solid lines = magnetic field; dashed lines = electric field.

Line modelling

A section of t.p.l. is shown diagrammatically in Fig. 1. In reel form this can be purchased commercially (e.g. from RS Components) but for line lengths of a few metres a t.p.l. may be made up by twisting together, uniformly, two pieces of PVC insulated wire (26 gauge, say) with a pitch of about 5 cm.

If we imagine the t.p.l. as laid along an x-axis perpendicular to the plane of this page, the field patterns that exist when equal-magnitude direct currents flow into the page at 'a' and 'b' respectively are shown in Fig. 2, where the solid lines indicate the nature of the magnetic field and the dashed lines the configuration of the electric field. These field patterns correspond also to those of the basic propagation mode for line transients discussed throughout this article.

The magnetic flux linking the wires is proportional to the current. The flux per unit current is represented by a series-inductance L per unit length. L is a parameter dependent on conductor geometry and can be estimated by analytical principles well known in field theory but a knowledge of L by itself is rarely required by t.p.l. users and, if needed, is best inferred from other readily measurable parameters. The electric field and flux associated with the conductors and the line charge on them are proportional to the p.d. between them, so the t.p.l. has also a per-unit-length capacitance C. As with L, this can be estimated theoretically if required but is readily determined practically.

Series losses may be represented by a per-unit-length resistance R and shunt losses resulting from leakage through wire insulation by a per-unit-length conductance G. The t.p.l., although distributed in nature, can nevertheless be considered as made up from as large a number as we wish of tiny lumped sections, each of length δx, connected in series. The idea of using a large number of small discrete lumps to simulate a continuous variable is not unfamiliar in electronics. Thus, a digital time base for an oscilloscope based on a counter and D-A converter produces a horizontal pattern of dots on the

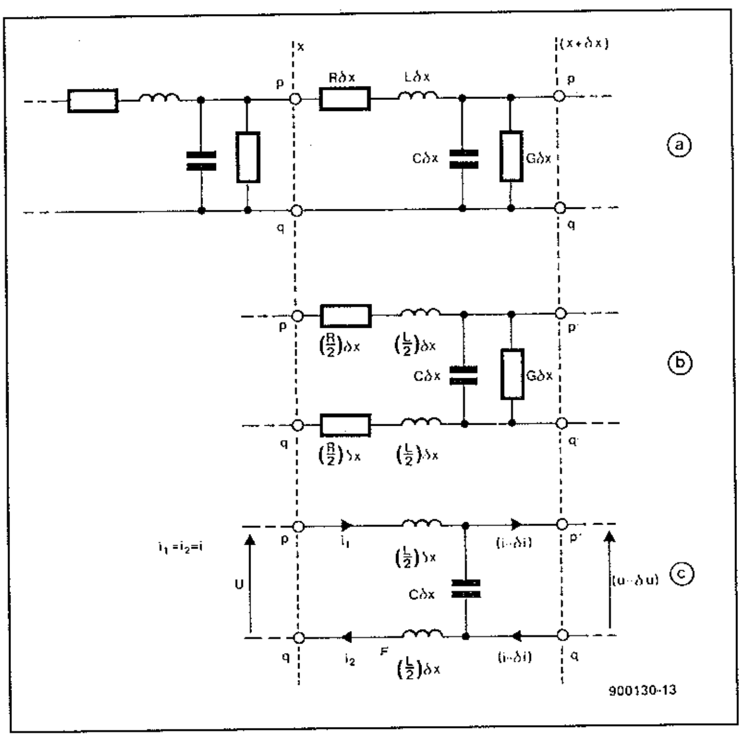

Fig 3. (a) a t.p.l. made up from lumped L-shaped sections; (b) equivalent form for (a); (c) reduced form for (b) for lossless line [$R = G = 0$].

screen. However, for a 10-bit converter, the number of dots exceeds 1000 and on a 10-cm screen the trace appears continuous.

The specific configuration of series and shunt components adopted to model an elemental section of line is a matter of sensible choice. All choices must by definition be equivalent in electrical characterization. We could use a 'T-section', but the 'L-section' shown in Fig. 3(a) is analytically more convenient.

Figure 3(a) is often used for coaxial cable lines in which the outer conductor is 'earthed' but this can be misleading, particularly for a t.p.l. because it may give the false impression that one of the conductors behaves in a different way electrically from the other. The alternative model shown in Fig. 3(b) shows R and L as equally shared between the two conductors of the t.p.l. and in that respect is conceptually more attractive.

For a t.p.l. a few metres long series and shunt losses can usually be neglected and the section reduces to the 'ideal' or 'lossless' form ($R = G = 0$); it is tempting to say that this is 'fortunate' for were it not so the t.p.l. would be of very restricted use.

For this case the relevant equations lend themselves simply to pictorial interpretation and the essential features of line operation are not obscured by second-order effects.

Line equations

Consider the section shown in Fig. 3(c). The currents, i_1, i_2, shown flowing in the upper and lower inductance elements must be equal in magnitude to i, say.

The reason for this is as follows. If we imagine the line to the right of the points p and q to be contained within the 'black box', the Law of Conservation of charge requires that $\int(i_1 - i_2)dt = 0$. This is true only, irrespective of the timescale t, if $i_1 = i_2 = i$.

Applying Kirchhoff's Voltage Law for loop voltage drops

$$u = (u_L/2) + (u + \partial u) + (u_L/2),$$

where u_L, the inductive voltage drop, is given by

$$u_L = (L\partial x)(\partial i/\partial t).$$

Substituting for u_L and rearranging:

$$-(\partial u/\partial x) = L(\partial i/\partial t) \qquad [1]$$

In passing, it may seem contrary to write the p.d. at $(x + \partial x)$ as $(u + \partial u)$ with a plus sign for the increment, when physical considerations tell us that it must be less than u. However, this is in the tradition of differential calculus. The physics of the problem gives the negative sign in [1].

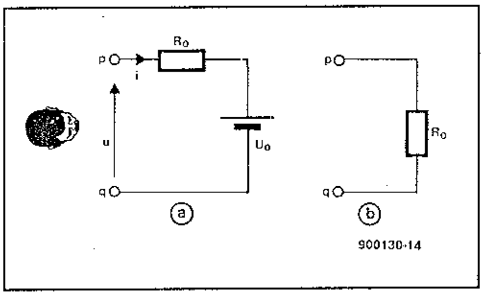

Fig. 4. View of line looking right between p and q at $t = 0$; (a) intitally charged line; (b) initially uncharged line

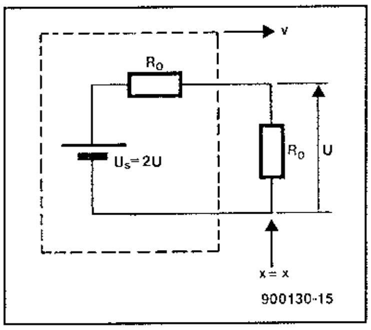

Fig 5. Sliding source description of step progress

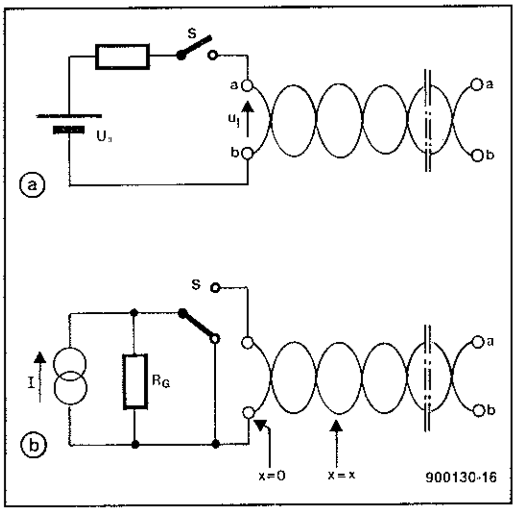

Fig 6. Applying a step input to a line: (a) voltage-step drive (Sw closes at $t = 0$); (b) current step drive

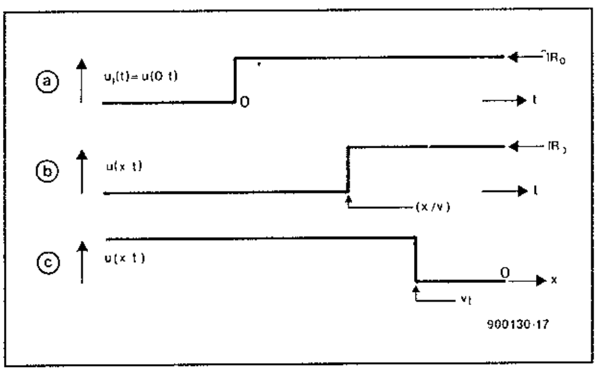

Fig 7 Voltage sketches for Fig 6(b)

Kirchhoff's Current Law for Fig. 3(c) gives

$$i = (i + \partial i) + (C\partial x)\{\partial(u + \partial u)/\partial t\}$$

For $\partial u \ll u$, a condition always achievable if ∂x is small enough, this reduces to

$$-(\partial i/\partial x) \approx C(\partial u/\partial t) \qquad [2]$$

In the limit case $\partial x \rightarrow 0$, $\partial t \rightarrow 0$ the approximation sign becomes an equality symbol. We have not proceeded to this limit yet, because the aim is to avoid the distraction of partial differential relationships that arise when a function is dependent on two or more variables. Indeed, combining [1] and [2] to eliminate ∂i or ∂u leads to the (partial differential) wave equation for an ideal line, but such a procedure requires us to solve the equation or, at least, quote solutions for it.

An alternative approach is to show that a voltage step at the input to the line travels along it with constant amplitude and uniform velocity. To do this we must first establish a relationship between u and i and then derive an expression for step velocity that is independent of x.

u/i relationship: characteristic resistance, R_0

Dividing each side of [1] by the corresponding side of [2] gives:

$$(\partial u/\partial i) = (L/C)(\partial i/\partial u)$$

or

$$(\partial u/\partial i)^2 = (L/C)$$

Taking the square root and proceeding to the limit

$$(du/di) = \sqrt{(L/C)} = R_0, \text{ say} \qquad [3]$$

We are entitled to express [3] in total differential form because it is valid irrespective of t. Equation [3] gives the limit case for small changes. The ratio is given the symbol R_0, because $\sqrt{(L/C)}$ has the dimensions of resistance. R_0 is known as the 'characteristic resistance'. It is characteristic of the line alone and not dependent on the nature of u or i, and is the incremental resistance looking to the right (or left) between terminals p and q, or p' and q'.

The expression 'characteristic impedance' is often used but is unnecessary for a lossless line. It conjures up thoughts of the frequency variable ω (or jω) and we are operating here strictly in the time domain.

R_0 is unlike a normal resistor in that it dissipates no power: it is a parameter, dependent on line geometry, that fixes a relationship between the instantaneous changes in i and u either of which can be regarded as a stimulus while the other is regarded as a response. In particular, a step change in u produces a step change in i and vice versa.

Integrating [3], the instantaneous value of u is:

$$u = iR_0 + U_0 \qquad [4]$$

Equation [4] is illustrated in Fig. 4 in which U_0 is any initial line voltage, shown here arbitrarily as positive, and for changes in u and i is accessible only via the series resistor R_0. This accessibility to a line voltage source only via a series resistor R_0 is true also looking to the left at a point on the line, because the line has no built-in directional properties for pulse propagation.

For an initially uncharged line, treated from now on, $U_0 = 0$ and the circuit looking to the right between p and q reduces to the simpler form of Fig. 4(b).

A step voltage of magnitude U appearing at one moment between p and q appears at a later time between p and q', charging up the line as it progresses with velocity v. There is no loss in amplitude as there are assumed to be no line losses.

Propagation velocity v

The propagation velocity, v, is found as follows. Multiplying each side of [1] by the corresponding side of [2]:

$$\{(\partial u)(\partial i)/(\partial x)^2\} = LC\{(\partial u)(\partial i)/(\partial t)^2\}$$

Thus, in the limit,

$$v = (dx/dt) = 1/\sqrt{LC} \qquad [5]$$

Since v is independent of x, the velocity is constant along the line. The time t_u is

$$t_u = 1/v = \sqrt{LC} \qquad [6]$$

Let us check [5] another way. We assume that v is constant and apply the principle of charge conservation. If a step wavefront U travels from $x = 0$ to $x = x$ in a time $t' = (x/v)$, the charge supplied to the line by the source is $i(x/v) = (U/R_0)(x/v)$. This must equal the charge accumulated by the line capacitance from $x = 0$ to $x = x$ and this is CxU. Thus,

$$(Ux/R_0 v) = CxU$$

and

$$v = 1/CR_0 \qquad [7]$$

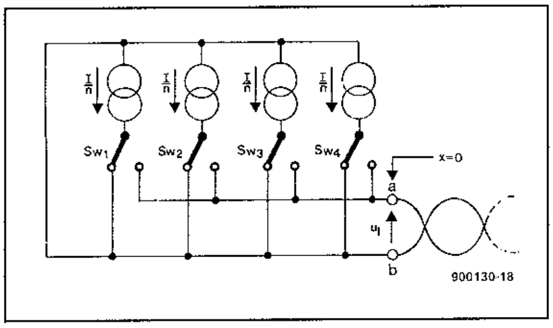

Fig. 8. T.p.l. with multiple step current drive.

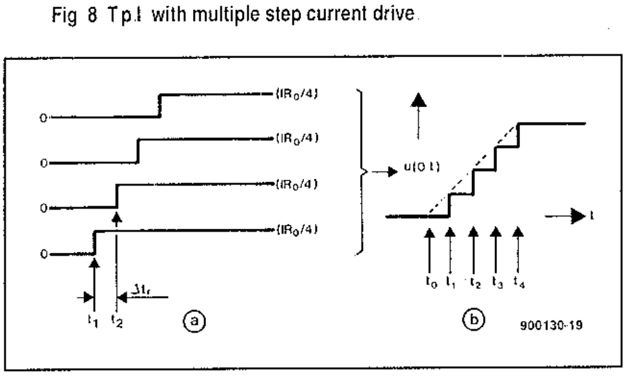

Fig. 9. (a) input voltage contributions for Fig. 8 for $n = 4$; (b) resultant staircase voltage input

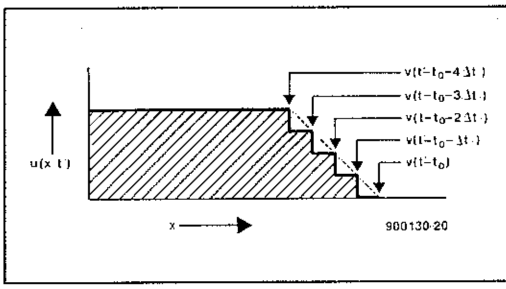

Fig. 10. $v(x;t')$ derived from Fig. 9(b)

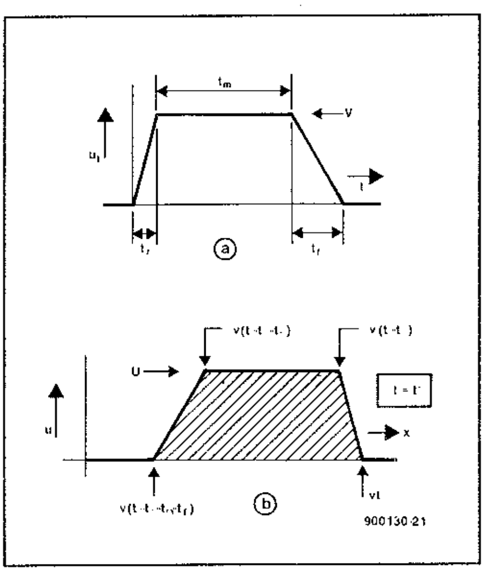

Fig. 11. (a) digital input signal to t.p.l.; (b) $u(x, t)$ derived from (a)

Substituting for R_0 (from [3]) in [7] gives the same value for v as in [5].

Note that R_0 and t_u are two basic parameters required to be known of a line. From these can be found C and L, if needed, with the aid of equations [3] and [6].

Models for step waveform progress

Progress of a step waveform is so basic that it merits further study. We consider a mechanical analogy and an electric circuit model.

In a mechanical analogy, we may consider a stationary hopper containing sand over a conveyor belt that is moving to the right. At a chosen moment, the exit pipe from the hopper is opened suddenly. The result is a constantly lengthening, uniform-thickness trace of sand on the belt. Sand here is, of course, analogous to electric charge.

A model for step progress attractive to the engineer more at home with lumped circuit theory involves the concept of a 'sliding source'. Consider the progress of a step voltage wavefront of magnitude U along an initially uncharged line in the direction of increasing x. Looking to the right at any point x, the remainder of the line appears as a resistor R_0 as shown in Fig. 5. Looking backwards towards $x = 0$, the line appears as a voltage source U_s accessible via a source resistor R_0 (inside dashed rectangle), both of which appear to slide along the line with velocity v. To produce a step of magnitude U at $x = x$, it is obviously necessary that $U_s = 2U$.

This sliding source approach is helpful in calculating what happens at the end of a line of finite length l.

Line voltage u (x,t): step and pulse drive

To investigate experimentally a t.p.l. subjected to a step input, the line can be 'voltage-driven' or 'current-driven' as shown in Fig. 6. In both cases, the condition of switch S is assumed to change at $t = 0$.

Simple experimental predictions of terminal voltage behaviour based on Fig. 6(a) require a knowledge of R_0 and the certainty of its constancy over the range of the output voltage swing. It is not possible to guarantee constancy using standard saturated transistor logic circuits (e g TTL) to voltage-drive the line.

In the current-drive scheme of Fig 6(b) the output resistance R_g is generally much greater than R_0 and can be ignored by comparison with it. This is the case, in practice, with a switched long-tail pair driver stage. $u(x, t)$ denotes line voltage as a function of variables x and t. Of special interest are: $u(0, t)$ the variation with t at $x = 0$, i.e. the input waveform, $u_i(t)$; $u(x', t)$ the waveform at an arbitrary point $x = x'$; $u(x, t')$ a plot of line voltage as a function of x at a specific time $t = t'$.

In Fig 7(a) $u_i(t)$ is a step of magnitude IR_0 because the line appears initially, at its input terminals, as a pure resistance R_0.

$u(x', t)$ in Fig. 7(b) is $u(0,t)$ delayed by a time interval (x'/v): the line is uncharged at x' till the step reaches that point. If the switching action occurs at $t = t_0$, the line is charged up to the point $v(t'-t_0)$.

Unlike $u_i(t)$ and $u(x', t)$, which can be monitored, $u(x, t')$ is not a waveform. However, if we choose an appropriate scale on the paper as in Fig 7(c) we can make the graph appear complementary to that of Fig 7(b). This means that the sum of ordinates of the two graphs at a given point on the horizontal axis gives a constant value. This scale changing 'trick' is useful in deriving $u(x, t)$ from $u_i(x, t)$ for the general case of a line signal that is not a step as we will show now.

In Fig 8 n current sources each of strength I/n are connected to a t.p.l. via switches S1–Sn.

S1 changes state at $t = (t_0 + \Delta t_r)$, where $\Delta t_r = \{(t_n - t_0)/n\}$, and pumps a current I/n into the line. This is followed at successive time intervals Δt_r by S2–Sn, respectively, causing additional current steps I/n to be applied in sequence to the line input.

Equation [4] specifies a linear relationship between u and i, so the Principle of Superposition is applicable and we can add algebraically the effects of each input taken separately to obtain the overall response.

The resulting waveform for $u_i(t)$ is a voltage 'staircase', which, for $n = 4$, is shown in Fig 9(b). The dashed line joining the edges of the treads intersects the t-axis at $t = t_0$. Suppose now that instead of $n = 4$ we let $n \to \infty$. The staircase edge then assumes the profile of the dashed line in Fig 9(b) and Fig 10. We have thus deduced $u(x, t')$ for a ramp input. The general case for an input of arbitrary shape is worked out similarly by considering steps of unequal magnitude and—if necessary—opposite polarity when the switches in Fig 8 change state.

Thus the digital input signal of Fig 11(a), with transition times t_r and t_f purposely chosen unequal, produces $u(x, t')$ in Fig 11(b). Figure 11(c) may be regarded as a scaled mirror image of Fig 11(a) displaced along the horizontal axis. An alternative graphical method for obtaining $u(x, t)$ from $u(0, t)$ is given in the reference at the end of this article.

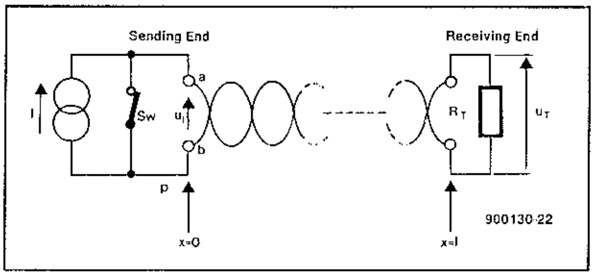

Fig 12 Current-driven terminated line.

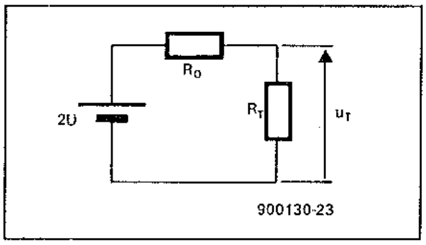

Fig. 13 Calculation of terminal voltage at $t = t_d$

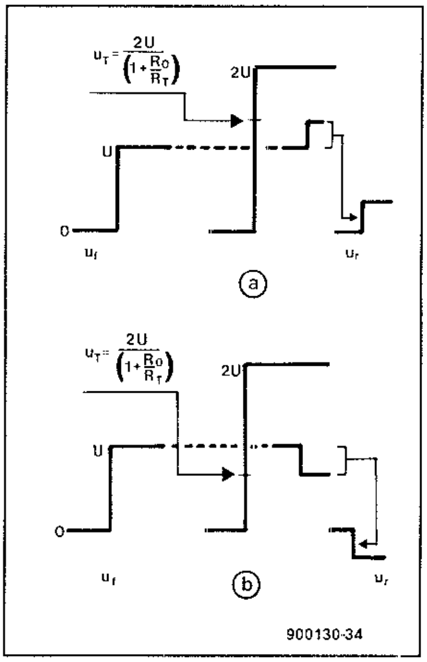

Fig. 14 Generation of v_r for $R_T \neq R_0$: (a) $R_t > R_0$; (b) $R_T < R_0$

Reflections

It is convenient to imagine a semi-infinite line, stretching from $x = 0$ to $x = \infty$, in an initial discussion of lines because it simplifies the presentation. However, once the progress of a step wavefront is understood we can consider what happens at the end of a line of finite length l when a pulse edge or a pulse of arbitrary shape reaches it.

Consider the scheme shown in Fig 12 where R_T is a terminating resistor. A voltage wavefront u_f of amplitude $U = IR_0$, which we call the forward wavefront, starts down the line at $t = 0$ when S opens. It reaches the end of the line in the one-way delay time $t_d = l/v = lt_u$.

The terminal voltage u_T at $t = t_d$ is calculated from the sliding source equivalent circuit of Fig 13: $u_T(t) = 2U R_T/(R_T + R_0)$. Now, $u_T = u_f$ (the terminal voltage step is equal to the amplitude of the forward voltage wavefront on the line) if $R_T = R_0$.

This is the case of a line 'matched' or 'correctly terminated' at the receiving end. Then R_T dissipates energy at the same rate as it is supplied to the line from the source. No energy is reflected, that is, sent back to the source. As far as any effect on the sending end is concerned, the line may just as well be considered semi-infinite despite its actual finite length. There is an analogy here in radar. If the energy in a radar beam is completely absorbed by a target, there is no reflection, that is, the target is 'invisible'. As far as the radar receiving equipment is concerned, the target may be regarded as located at a point an infinite distance away. Suppose, however, that $R_T \neq R_0$. Then $u_T \neq u_f$, all the energy associated with u_f cannot be absorbed by R_T, and a reflected wavefront u_r is pro-

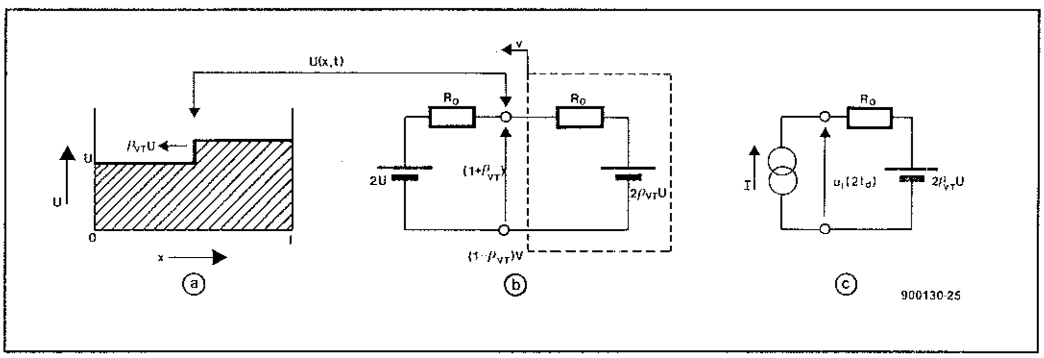

Fig. 15 (a) line voltage for $2t_d \geq t > t_d$; (b) sliding circuit equivalent circuit form for (a); (c) circuit for calculating $v_i(2t_d)$

duced. The amplitude and polarity of u_r must be such that the Principle of Superposition is applicable at the termination. Thus.

$$u_i + u_r = u_l$$

or

$$u_r = u_l - u_i \quad [9]$$

Substituting for u_l from [8] and $u_i = U$ gives

$$u_r = [\{2UR_T/(R_T+R_o)\} - U] = \rho_{VT} U \quad [10]$$

where ρ_{VT} is the voltage reflection coefficient at the termination and is defined by

$$\rho_{VT} \equiv (R_T - R_o)/(R_T + R_o) \quad [11]$$

Figure 14 shows a geometrical construction giving u_r for the cases:

(a) $R_T > R_o$ and hence $\rho_{VT} > 0$ and

(b) $R_T < R_o$ and hence $\rho_{VT} < 0$

For either condition the reflected voltage wavefront travels back to the source.

A plot of line voltage for $2t_d \geq t > t_d$ is shown in Fig. 15(a) for $\rho_{VT} = 0$. This results from adding u_r to the existing line voltage giving a total line voltage $(1 + \rho_{VT})U$ at the position of the wavefront.

The total line voltage is also obtained from the sliding source equivalent circuit which, in this case, comprises a generator $2\rho_{VT}U$ in series with an output resistance R_o as shown in Fig. 15(b):

$$u(2t_d) = IR_o + 2\rho_{VT}U = U(1 + 2\rho_{VT}) \quad [12]$$

Since there is already a line voltage U and $u_r = \rho_{VT}U$ this means a further forward, reflected wavefront of amplitude $\rho_{VT}U$. This also follows from [11] since the voltage reflection coefficient is unity for an ideal current source.

The current-driven line of Fig. 12 with $R_I \neq R_o$ is of restricted use. Two cases of reflection of practical interest for a current-driven line with an intentional mismatch at the receiving end are considered next.

With reference to Fig. 16, in which a shunt matching resistor is incorporated at the sending end, the two cases correspond to $R_T = 0$ and $R_T = \infty$.

Consider first the case $R_T = 0$. Writing U for $IR_o/2$, it follows that $u_l(0+) = U$. From [11], $\rho_{VT} = -1$. The equivalent circuit for calculating $u_l(2t_d)$ and $u(x)$ for $t > 2t_d$ is shown in Fig. 17.

In Fig. 18 $u_l(t)$ is a pulse of amplitude U and duration $2t_d$. The line input current, i_l, and the energy supplied by the source W_s, are shown in Fig. 18(b) and Fig. 18(c) respectively.

An argument based on the Principle of Conservation of Energy leads to an algebraic expression for t_u. Thus

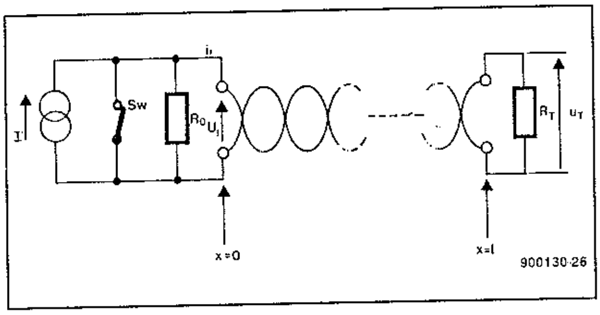

Fig 16 Current-driven line, matched at the sending end.

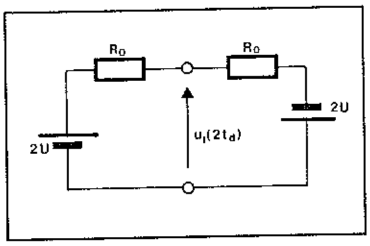

Fig. 17. Equivalent circuit for calculating $v_l(2t_d)$ for $R_T = 0$

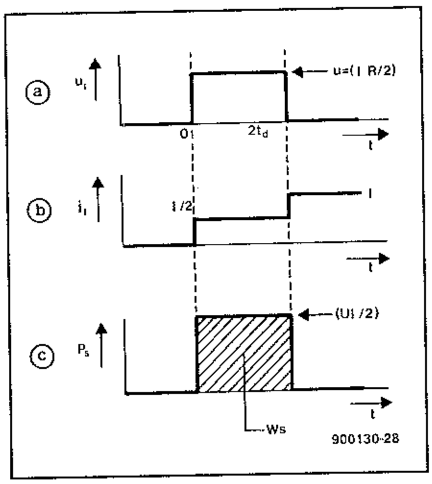

Fig 18 For $R_T = 0$, the energy, W_S, supplied to the line by the source (c) is obtained by multiplying graphs (a) and (b)

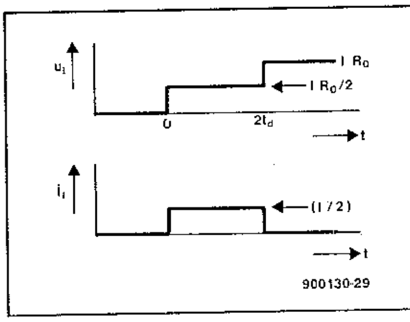

Fig 19 $v_l(t)$, $i_l(t)$ for the case $R_T = \infty$

$$W_s = (IU/2)2t_d = IU = (I)^2R_o/2 \quad [13]$$

At $t = 2t_d$ the line stores no energy in its electric field since at that time $u_l = 0$. All the energy W_m is stored in the magnetic field:

$$W_m = L l(I)^2/2 \quad [14]$$

Equating W_m and W_s yields:

$$t_u = (t_d/l) = (L/R_o) \quad [15]$$

However, $R_o = \sqrt{(L/C)}$ so that

$$t_u = \sqrt{(LC)} \quad [16]$$

as previously shown in [6].

With reference to Fig. 16, the case $R_T = \infty$ gives the waveforms for $u_l(t)$ and $i_l(t)$ in Fig. 19.

Conclusion

This article has dealt in detail with some aspects of line pulse operation that are either ignored or skimpily covered in the literature.

Reference:

Digital Signal Transmission Line Circuit Technology, by B.L. Hart. Van Nostrand Reinhold (UK). 1988 (Chapter 3) ■