A SIMPLY ELEGANT *L*-*C*-*R* BRIDGE

The balanced bridge described in this article measures capacitance from 1 pF to 10 μF, resistance from 1 Ω to 10 MΩ, and inductance from 1 μH to 100 μH. Ideal for checking the values of non-marked or otherwise non-identifiable components, the instrument costs next to nothing, and can be built from parts from the junk-box.

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THE first design of the instrument proposed here was a basic capacitance-measuring bridge which consisted of nothing more than two capacitors, a 10-kΩ carbon linear potentiometer, a crystal earpiece, and a total of four solder joints, as shown in Fig. 1.

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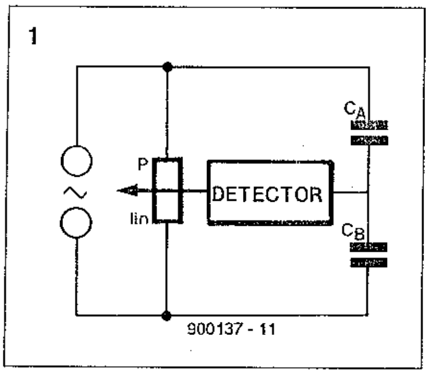


An alternating voltage at audio frequency was required to feed the bridge, and a quick glance around the author's radio shack showed a number of possible sources, e.g., an oscilloscope which offered a 1-kHz square-wave signal at 1 V_{pp} on its front panel for self-calibration, an RF signal generator which offered a 1-kHz sine-wave at a few volts rms, a home-made AF oscillator with a 2-V output and, if the worst came to the worst, the audio output from the transistor broadcast radio tuned to the pop-music channel!

All were tried and found to provide a fully audible signal in the crystal earpiece, and rotation of the balance potentiometer spindle yielded a clearly discernible and sharp null, more than adequate to afford repeatable accuracy of measurement.

The capacitors were replaced with resistors, with equally promising results for the measurement of resistance.

But what of inductance in the RF range? A few turns of enamelled copper wire were wound on to an available 7-mm former with an iron-dust slug core. A 100-pF capacitor was soldered across the coil. Its resonant frequency was adjusted to about 15 MHz with the aid of a gate dip oscillator. Next, the inductance of the coil was calculated to be approximately 1 μH. That would be the 'unknown' inductor of low value.

Another coil was wound with about ten times as many turns to produce a higher value of inductance. The actual value was not important, as will be explained later.

With the two inductors connected into the bridge in place of the two capacitors, again it was possible to obtain a sharp audio-null to indicate balance.

Turning the core in the 1-μH coil to produce a different value of 'unknown' inductance required the potentiometer to be re-adjusted to restore balance. This demonstrated the viability of the simple bridge as a measuring device suitable for all three types of passive component, i.e., for inductance (*L*), capacitance (*C*) and resistance (*R*).

Balanced bridge for *L*-*C*-*R* measurement

Consider first the simple resistive potential divider shown in Fig. 2a. The potential difference, or voltage drop, across R_a is

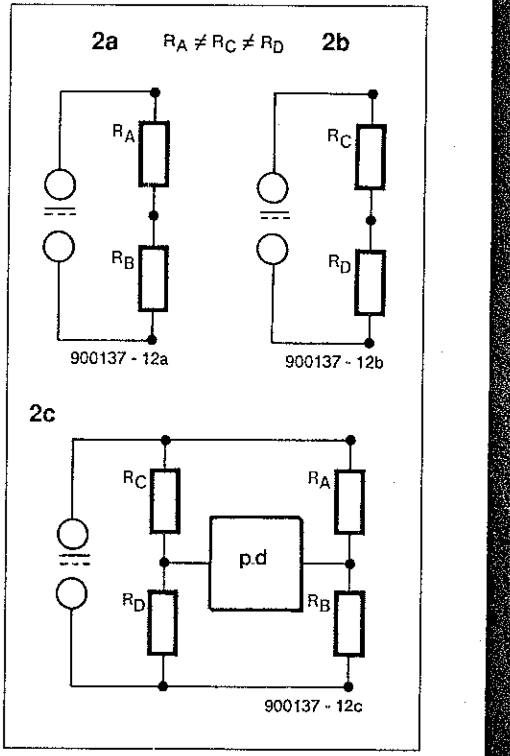
$$U_s \frac{R_a}{R_a + R_b}$$

where U_s is the supply voltage. Now consider the other resistive potential divider in Fig. 2b, which incorporates unspecified but different values of resistor from those in Fig. 2a. The potential difference, or voltage drop, across R_c is

$$U_s \frac{R_c}{R_c + R_d}$$

Connecting the two potential divider networks in parallel to form a basic resistive bridge, and feeding them both from the common voltage supply, as shown in Fig. 2c, does not alter the two equations given for the potential differences across R_a and R_c.

If, however, the voltage drop across R_a is different from that across R_c, then a voltage detector connected between junction R_a-R_b and junction R_c-R_d will indicate the difference between the two voltage-drop values. But, if the voltage drop across R_a is equal to that across R_c, the potential difference will be nought, and the voltage indicator will read zero or a voltage 'null'. The electrical bridge formed by the two potential divider networks R_a-R_b and R_c-R_d is then said to be balanced, and that will also be so irrespective of whether d.c. or a.c. is used for the voltage supply. The mathematical equation for such a null condition is given by



$$U_s \frac{R_a}{R_a + R_b} = U_s \frac{R_c}{R_c + R_d}$$

which simplifies to

$$\frac{R_a}{R_b} = \frac{R_c}{R_d}$$

If the value of R_b is known and fixed, and if the ratio R_c/R_d is known, it is possible under this null condition (i.e., when the bridge is at balance) to determine the actual value of an unknown resistor R_a from

$$R_a = R_b \frac{R_c}{R_d} = (\text{known } R_b) \times (\text{ratio } R_c/R_d)$$

For example, if R_c is made equal to R_d their ratio is 1.0, and the null condition becomes $R_a = R_b$, irrespective of the actual values of R_c and R_d. Only their ratio is of importance.

Further, if the ratio R_c/R_d is made adjustable and calibrated, and if the value of the known resistor R_b is made to be switch-selectable, the unknown resistor, R_a, can be quantified over a wide range of values.

Scale design: the basics

For convenience, R_c and R_d can be replaced by a linear-law rotary potentiometer of any convenient resistive value to be provided with a circular scale and a pointer knob. The scale is then calibrated to read the ratio of resistive values measured between the centre tag and the two outer tags of the potentiometer for different angles of spindle-rotation.

At mid-travel, for example, the resistance measurements between the centre tag and the two outer tags would be equal in an ideal linear-law potentiometer. At three-quarters traverse, the resistance of one section would be some three times that of the other section, giving a ratio of 3:1 and vice-versa.

In other words, as the spindle is rotated away from mid-position in one direction, the ratio will increase from 1.0 upwards towards infinity, and in the other direction it will decrease from 1.0 towards zero.

Although the rotational movement of a standard linear-law carbon potentiometer is restricted to about 300 degrees of travel, fortuitously and very conveniently the ratios ×10 and ×0.1 fall at approximately 90 degrees on either side of mid-travel. So, if mid-travel is positioned at the top of the scale, i.e., at 360 degrees, then the dial can be marked ×0.1, ×1.0, and ×10 at 270, 360 and 90 degrees respectively, and ×0.01 and ×100 at 240 and 120 degrees respectively; to a first approximation.

The value of a resistor is thus readily determined by rotating the potentiometer until a null is detected. The ratio indicated on the scale is subsequently multiplied by the value of the known resistor, R_b. Mathematically,

$$R_a = R_b \times [\text{ ratio } R_c/R_d \text{ at null }]$$

By assigning a selection of different values for the *known* resistor R_b, the range of measurement for the *unknown* resistor, R_a, can be conveniently modified. To simplify measurement even further, it is preferable to use whole number values for the *known* resistors, e.g., 100 Ω, 10 kΩ, 100 kΩ etc. These known resistors, which for convenience can be range-selectable, are then renamed *range* resistors, and the switch escutcheon is marked with their values. An example: if a null occurs at, say, ×0.05 on the 1,000 Ω range, the unknown resistor has a value of (1,000 × 0.05) = 50 Ω.

Inductance measurement

The above principle of measurement can also be applied to inductance, with resistors R_a and R_b replaced by inductors L_a and L_b. To be effective, the bridge *must* be fed with an a.c. voltage at a frequency, *f*, high enough to produce inductive reactances X_a and X_b respectively, sufficient to provide potential differences suitable for null detection.

The simplified potential difference equations at null condition are:

$$X_a = X_b \frac{R_c}{R_d}$$

where X for inductive reactance is

$$X = 2\pi f L$$

So, this equation simplifies to:

$$L_a = L_b \frac{R_c}{R_d}$$

which conveniently uses the same ratio multiplier R_c/R_d as for resistance measurement, hence the same ratio scale can serve for both resistance and inductance.

Capacitance measurement

For the measurement of capacitance with inductors L_a and L_b replaced by capacitors C_a and C_b, again the voltage supply must be a.c. to provide, in this case, capacitive reactances X_a and X_b.

The simplified equation for potential difference in the null condition is

$$\frac{X_a}{X_b} = \frac{R_c}{R_d}$$

where X for capacitive reactance is

$$X = \frac{1}{2\pi f C}$$

$$X_a = X_b \frac{R_c}{R_d}$$

which simplifies to

$$C_a = C_b \frac{R_d}{R_c}$$

Note that in this expression the ratio multiplier, R_d/R_c, is inverted with respect to the one used for the resistance and inductance measurements. In practice, this means that a mirror-image of the ratio scale is required for the measurement of capacitance. So, the scale

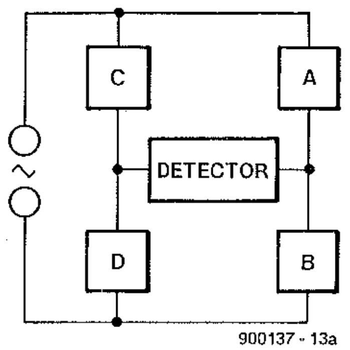
0.01 0.1 1.0 10.0 100.0

becomes

100.0 10.0 1.0 0.1 0.01 etc

Again, a selection of range capacitor values for C_b will provide the desired range of measurement, and for user convenience,

3a



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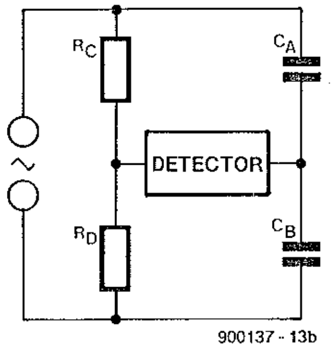

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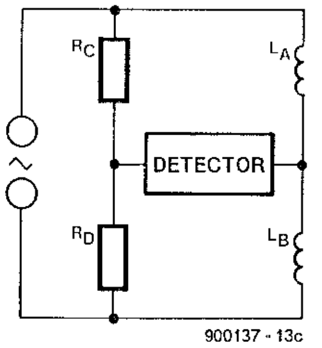

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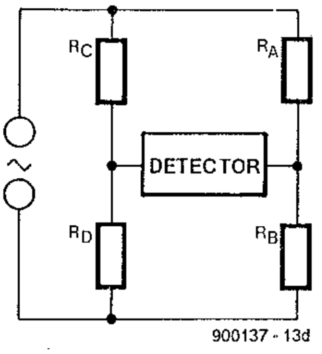

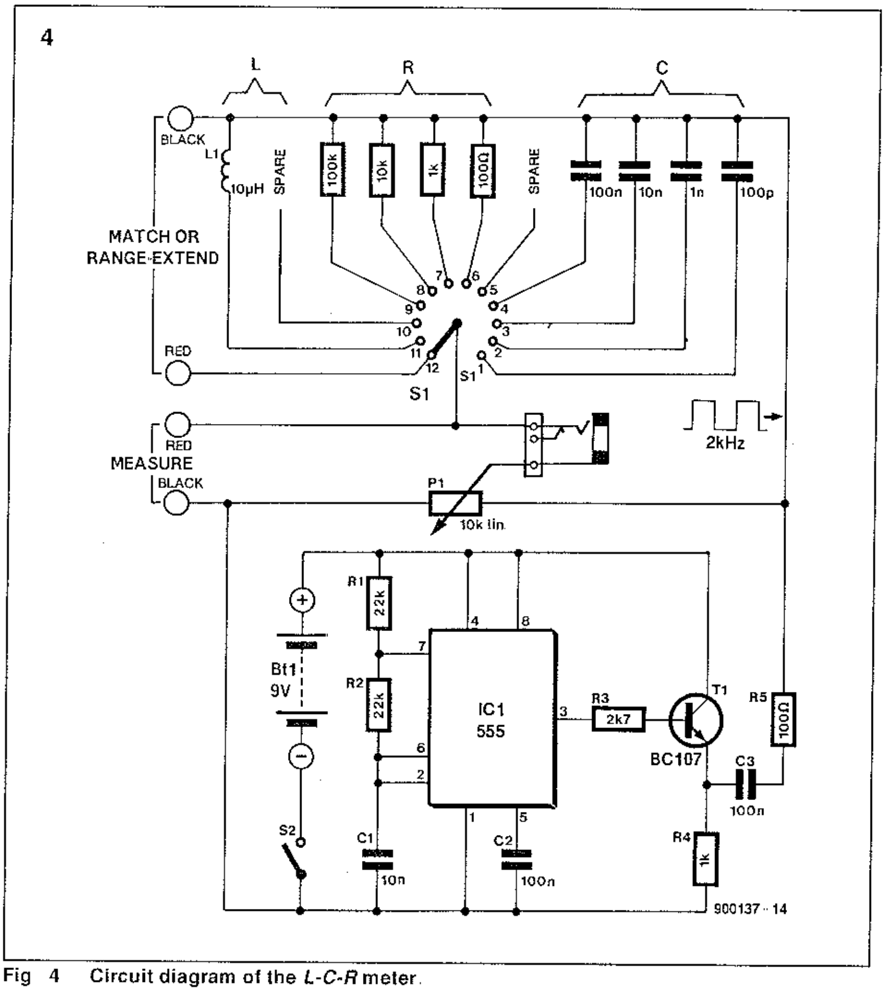

Fig 4 **Circuit diagram of the *L-C-R* meter.**

they should be round numbers, e g , 100 pF, 0 1 µF, etc

Circuit design concept

The circuit diagram of the *L-C-R* bridge is given in Fig 4. A rectangular wave of about 9 V_{PP} is provided by a 555 IC1, configured as an astable multivibrator, with a repetition frequency of about 2 kHz.

The 2-kHz output from the astable is buffered by an emitter follower transistor, T1, to minimize loading of the 555 output circuit by the bridge when this is switched to the lower resistance/reactance ranges.

A crystal earpiece is used for the null detector. Its high impedance offers a better audible signal than would an electromagnetic version to help with the determination of the null, particularly when measuring inductance

Accuracy of measurement depends mainly on the accuracy of the component values switched into the circuit by the *L-C-R* range switch, S1, and on the quality of the linear-law potentiometer, P1, used for the balance control

The prototype of the *L-C-R* bridge was built with a standard off-the-shelf linear carbon potentiometer for the balance control (a 1,000-Ω version should serve equally well), low-tolerance, high-stability resistors and capacitors, and a commercially available moulded RF inductor of 10% tolerance

It was considered that a single 10-µH range inductance would provide a wide enough range of inductance measurement for normal RF purposes, but provision has been made for extending the measuring range for each type of component This can be provided permanently by wiring an additional L_b, C_b, or R_b on to the range switch, or temporarily by connecting an appropriate component across the MATCH terminals with the RANGE switch in the 'match' position.

The MATCH terminals serve also to allow value matching of a pair of external components When the two are exactly equal in value, the null falls exactly at the '1 0' position on the ratio scale, and when not matched, the scale indicates the relative value of the component connected to the MEASURE terminals compared to that at the MATCH terminals.

Although no d c polarizing voltage has been included for electrolytic or tantalum capacitors the bridge can be used for the measurement of such capacitors without problems.

Current drain from the 9-V PP3 battery is only about 7 mA

Construction

Because electrical shielding is not required, the components are housed in a low-cost plastic container with removable flat cover. Externally accessible components, i e , balance potentiometer, range switch, MATCH and MEASURE terminals, ear-piece socket and battery switch, are mounted on to the removable panel

One lead of each of the 'range' components is soldered directly to the appropriate lag-terminal of the RANGE switch, and the other lead of each component is soldered to a self-supporting ring of tinned copper wire

The RANGE switch is a single-pole 12-position rotary midget wafer type, with a plastic spindle and fixing bush, to minimize stray capacitance which might adversely affect the measurement at the lower picofarad range. A flat on the spindle allows a push-on knob to be used, of the type which has a moveable cover-cap to allow alignment of the pointer

COMPONENTS LIST

Resistors:

2	100Ω 1% metal film	1 for range switch 1 for calibration
2	1kΩ 1% metal film	1 for range switch 1 for calibration
2	10kΩ 1%, metal film	1 for range switch 1 for calibration
2	100kΩ 1% metal film	1 for range switch, 1 for calibration
2	22kΩ	R1;R2
1	2kΩ7	R3
1	1kΩ	R4
1	100Ω	R5
1	10Ω linear potentiometer	P1

Capacitors:

2	100pF 1% silvered mica	1 for range switch, 1 for calibration
2	1nF 1% silvered mica	1 for range switch, 1 for calibration
2	10nF 1% polystyrene	1 for range switch, 1 for calibration
2	100nF 5% polyester	1 for range switch, 1 for calibration
2	10nF ceramic	C1;C2
1	100nF ceramic	C3

Inductors:

1 10µH encapsulated polypropylene
1 10µH air-cored self-wound (see text)

Semiconductors:

1	555	IC1
1	BC107	T1

Miscellaneous:

1 enclosure, ABS plastic, 120×65×40 mm, with lid
1 1-pole, 12-way rotary midget wafer switch plastic spindle/bush
1 miniature on/off switch
1 3 5-mm panel mounting jack socket
1 crystal ear-piece
1 DIL socket 8-way
1 35-mm square section of 0.1-inch hole-spacing copper-strip board
2 4-mm terminal post red
2 4-mm terminal post, black
1 PP3 battery with twin press stud and holder

The drawing in Fig. 6 shows a suitable escutcheon for the L-C-R SELECT/RANGE switch.

The linear-law carbon potentiometer used for the balance control also has a plastic spindle and fixing bush, but preferably without a flat on the spindle to allow a grub-screw type of pointer knob to be aligned to the '1.0' mark on the ratio-scale.

The 555 integrated circuit, transistor T1, and the eight associated passive components are assembled on to a 35-mm square piece of 0.1-inch hole-spacing copper strip board. It is reasonable to use an 8-way DIL socket for the 555. The actual lay-out of the few components on the board is uncritical, hence does not warrant a guidance sketch. The finished 2-kHz oscillator board is small enough to be self-suspended by its connecting wires, but its PP3 battery may need an elementary fixing bracket or zip-strap.

Four 4-mm terminal posts for the MEASURE and MATCH pairs of terminals allow the components to be either loosely plugged in or more securely screw-fastened to suit the circumstance.

The size of the enclosure is not critical, indeed the prototype used a plastic box of about the same general shape and size as a standard wall-socket box of 75×75×45 mm, but a box with a panel of, say, 65×120 mm would accommodate a circular scale of readable dimension plus the range switch escutcheon, with room to spare for the terminals, battery switch and ear-piece socket.

Calibration

The ratio scale needs to be calibrated for optimum results, because the linear-resistance characteristic of the chosen balance potentiometer may not necessarily be the same as that used in the author's prototype. The sample scale shown in Fig. 5 may be used for guidance.

Although the standard potentiometer has a rotational travel of about 300 degrees, it is recommended that the usable ratio scale be confined to ±90 degrees about centre (ratios: ×0.1, ×1.0, ×10), or at most ±120 degrees (ratios ×0.01, ×1.0, ×100).

The easiest way to calibrate the scale is to remove the pointer knob and temporarily affix a circular paper-scale centrally over the fixing bush, with the '1.0' mark uppermost at 360 degrees.

The circular scale should have two circular bands, one marked 'R/L' for resistance and inductance, and the other marked 'C' for capacitance.

Plug in the ear-piece, select the 100 Ω range and switch on the oscillator. The parts list recommends the availability of four duplicate low-tolerance high-stability resistors. These will be used as the external standards for calibration/test of the scale.

Connect the 100 Ω resistor to the MEASURE terminals, and carefully rotate the balance pot spindle until a null is obtained in the ear-piece. Loosen the grub-screw in the pointer-knob, and carefully position the knob on to the spindle with its pointer exactly at '1.0' (360 degrees) position of the scale, taking care not to move the spindle. Tighten the grub-screw, and rotate the knob to either side of '1.0' to check that the null is still coincident with the '1.0' mark.

Select the 1,000 Ω range and, with the 100 Ω resister still connected, adjust the pointer for null in the ear-piece. This should occur at about 270 degrees. Mark the R/L scale '×0.1' at that position (i.e., 1,000 Ω × 0.1 = 100 Ω).

Switch to the 10 kΩ range, when the null should appear at about 240 degrees, and mark the R/L scale '×0.01' at that position (i.e., 10 kΩ × 0.01 = 100 Ω).

Replace the 100 Ω resistor by the 1,000 Ω resistor and, with the RANGE switch set to 1,000 Ω, check that the null is at '1.0' (360 degrees).

Switch to the 100 Ω range, still with the 1,000 Ω resistor, when the null should occur at about 90 degrees. Mark the R/L scale '×10' at that position (i.e., 100 Ω × 10 = 1,000 Ω).

Replace the 1,000 Ω resistor by the 100 kΩ resistor, set the range switch to 1,000 Ω, and the null should occur at about 120 degrees, and mark the R/L scale '×100' at that posi-

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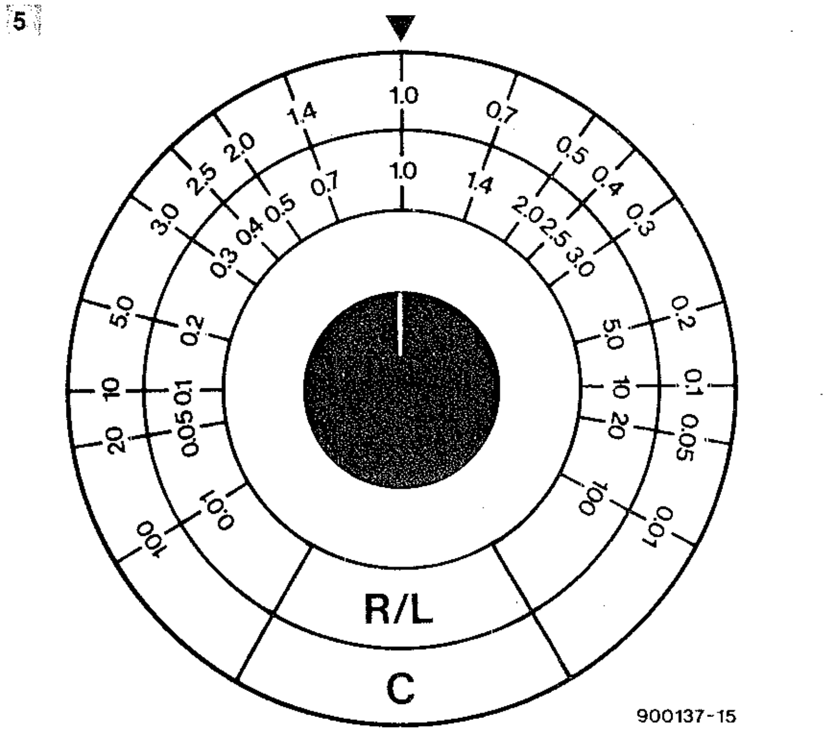

Fig. 5. Scale design for the balance control

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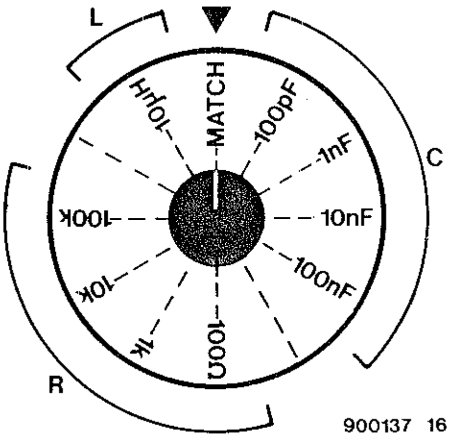

Fig. 6. Range switch escutcheon.

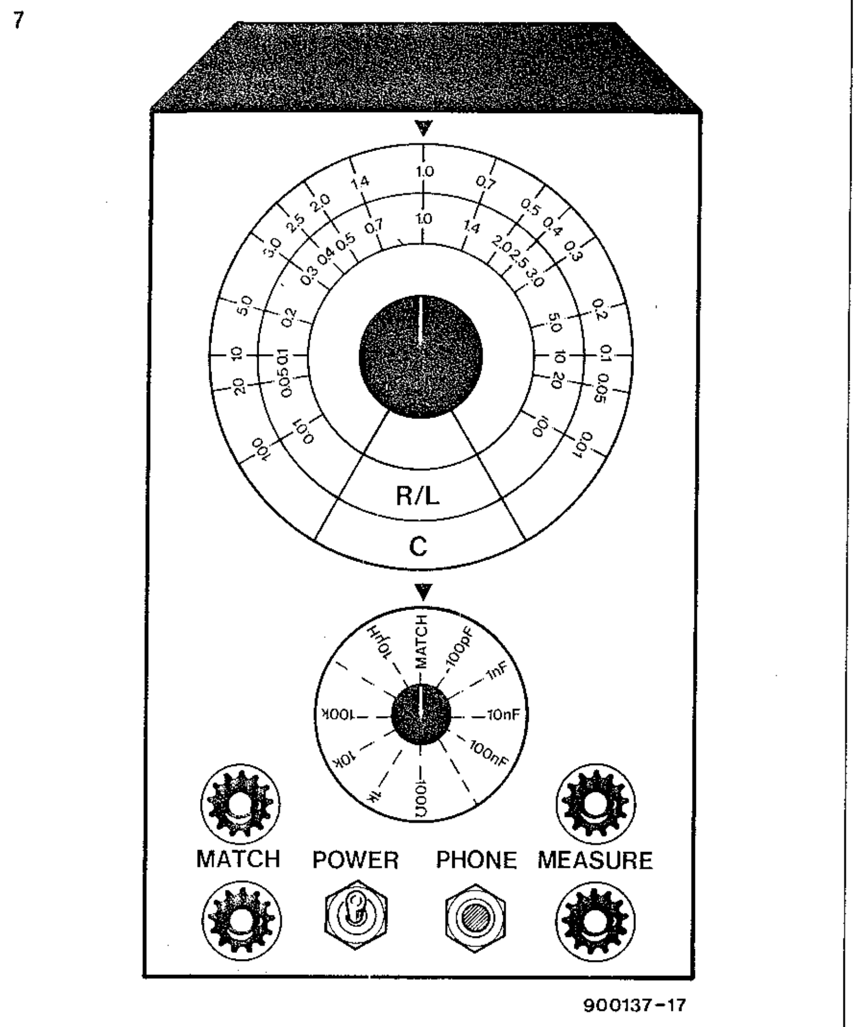

Fig. 7. **Suggested front panel layout of the *L-C-R* bridge**

R/L marking	C marking
0.1	10
0.2	5.0
0.3	3.3
0.4	2.5
0.5	2.0
0.6	1.7
0.7	1.4
0.8	1.3
0.9	1.1
1.0	1.0
0.01	100
0.02	50
20	0.05
80	0.013

tion (i.e. 1,000 Ω × 100 = 100 kΩ).

It will now be obvious that the ratios ×0.1 to ×1.0 and ×1.0 to ×10 each span about 90 degrees of travel, whereas the ratios ×0.01 to ×0.1, and ×10 to ×100, each span only about 30 degrees of travel. This means that calibration points beyond less than ×0.1, and greater than ×10, become increasingly cramped — but still very repeatable and acceptably accurate provided that care is taken with the calibration of the intermediate points in each sector.

Nulls are still very detectable even further towards the end-stops of travel, but the cramping is even more pronounced. That is where the advantage of a large diameter scale becomes apparent. The majority of *in vivo* measurements will however fall within the ±90 degree bands, where the scale is uncramped and clearly readable.

To calibrate the intermediate scale-points, it is advisable to restrict them to the 120 degree sectors on either side of '1.0', and to use whole numbers for the external calibration resistors rather than the decimal values of the 'preferred' series. The easiest way to do this is to use a 1,000 Ω standard linear-law potentiometer with a wire connected to its centre tag, and another to one of its outer tags, and to set it to a selection of values by means of an ohm-meter, i.e., 200, 300, 400 ... 900 Ω, etc. These values are then used to establish the intermediate scale points, i.e., ×0.02, ×0.03, ×0.04 ... ×0.09 using the 10 kΩ range; ×0.2, ×0.3, ×0.4 ... ×0.9 on the 1,000 Ω range; and ×2, ×3, ×4 ... ×9 using the 100 Ω range.

Similarly with values between 20 kΩ and 90 kΩ on a 100 kΩ potentiometer, to give scale points of ×20, ×30, ×40 ... ×90 using the 1,000 Ω range.

The resistance calibration scale applies equally to the measurement of inductance, but a mirror-image scale is required for capacitance measurement. The same calibration positions pertain, but the C scale must be marked with the inverse values from the R/L scale points rounded up for practical purposes, i.e., as shown in the table. Calibration is now complete, and the bridge is ready for use.

Does it work?

Yes. Results when measuring resistance can be accepted with confidence, as can measurements of capacitance, provided that the capacitors being measured are of good electrical quality. However, regarding the measurement of inductance, it must be borne in mind that the balance equations have been simplified by assuming zero resistance in the small values of inductance to be measured.

In practice, this will not be the case, depending on the construction of the particular RF coil. For example, the moulded 10 μH inductor used in the prototype has a series resistance of about 0.3 Ω owing to the very thin wire used for the coil, and whilst this does not unduly detract from the calibration accuracy when measuring inductors wound with similarly thin wire gauges, it does create inaccuracies when trying to measure the values of coils with heavier gauge wires, e.g., 14 to 24 SWG (14 to 25 AWG).

The solution to the measurement of inductance of heavier gauge coils is nevertheless quite simple, by using the MATCH terminals with an alternative 'standard' 10 μH inductor connected across them, but wound from thickish wire. A suitable alternative 10 μH air-cored coil can be constructed easily by close-winding 24 turns of 20 SWG (1.0 mm dia., or 21 AWG) enamelled copper wire with a winding-span of about 24 mm on to a PVC former of 25 mm outside diameter; or 32-turns of 20 SWG wire by 32 mm span on to a 20 mm outside o.d. PVC former; or 69 turns by 69 mm span of 20 SWG wire on to a 12.5 mm former. In each case, allow 10 mm end-tails.

Measurement of thicker-wire unknown inductance now follows normal procedure, but with the switch set at its 'match' position instead of the 'L' position, and with the 10 μH thick-wire standard inductor connected to the MATCH terminals.

To get the feel of the bridge, try measuring a selection of *L*, *C* and *R* components, and the effect of tolerance on nominal values. Then try matching component values by connecting pairs of nominally equal values to the MATCH and MEASURE terminals until null is obtained at '1.0' on the scale. Try also extending/modifying the range of measurement by connecting a known value component to the external MATCH terminals. And for interest only, try measuring the relative effect of an iron-dust slug versus a brass slug in a low-value RF inductance. ■