Looking into impedance

Cyril Bateman provides solutions for measuring a component's impedance – regardless of frequency.

Impedance is the ability of any circuit to oppose or limit the flow of current when stimulated by a suitable voltage source[1]. At the simplest level, it is the dc resistance value of a typical resistor, calculated from measurements of voltage drop and through current, using Ohm's law

With an ideal resistor, circuit current and applied alternating voltage would be exactly in phase with each other, so measured using dc then ac stimulus, both values would be identical. Accurate measurements being made simply by measuring voltage and current magnitude.

In practice, such an ideal component cannot exist Practical resistors possess both capacitive and inductive parasitic elements, so magnitude only measurements at ac or dc will differ. Parasitic elements cause the phase of the resistor's current to differ from the applied voltage So apart from those special frequencies when the component is self resonant, true measurement of the ac impedance of components[1] requires that phase difference as well as impedance magnitude be measured

Consider the measurement of capacitance and inductance. Neither parameter is directly measurable using ac test frequencies. The property actually measured is impedance, from which the effective capacitance or inductance and resistance values are calculated.

True impedance measurement of components at the relevant frequency is important for two reasons. Many circuit applications or simulations, require that component parameters be known at the end use frequency and secondly to permit correlation with the component manufacture end of line testing

Terms used

Using Ohm's law at ac, it is usual to replace the resistance term with the magnitude of impedance vector **Z**, hence $|Z|=|V|/|I|$. Including phase angle, the description for impedance in polar format becomes $|Z|\angle\theta=(|V|/|I|)\angle\theta$

> ### Measuring phase
>
> The phase difference of two waveforms, can be calculated by simply measuring the time difference at their zero crossing, dividing this time difference by the periodic time and multiplying by 360 This principle is used in many dedicated phase meters.
>
> This time difference can also be measured using a double beam oscilloscope with an X-trace readout like the PicoScope virtual dual channel oscilloscope that I have used for the measurements shown in this article.
>
> Audio Precision's System One is commonly used for audio measurements. It can be fitted with two voltage measurement channels and phase difference meter as an option Consequently, this test set could also make reasonably accurate impedance measurements, subject to the limitations of its 100kΩ input impedance

With electronic components, impedance values are usually expressed[2] in the rectangular format, $|Z|=R\pm jX$, in which R represents the in-phase resistive component, commonly called esr, or equivalent series resistance. Value X is the out-of-phase reactive component, i.e. capacitive or inductive reactance, of the measured vector.

Fortunately most pocket calculators include functions to easily convert impedances between polar and rectangular formats. From this basic rectangular format, many other mathematical transpositions are derived. Further details are in the panel entitled Impedance transpositions.

Sources of error

Measurements of high impedance, at ac or dc, can be influenced by the shunting effect of the input impedance of the voltmeter used

When measuring low impedances, the resistance, capacitance and inductance of any test leads used, can easily exceed the value to be measured. Consider the effects of the test leads when measuring dc resistance.

Conventional dc measurements. Resistance values greater than 100Ω, can be measured using almost any multimeter, with sufficient accuracy for most design needs. For values below 10Ω however, the measured value will be overstated, since the resistance of the test leads and contacts used, are included in the displayed result.

Using four terminals. When measuring resistance below 1Ω, lead resistance can exceed the value being measured. These errors resulting from test-lead resistance, can be avoided[1] by using the four-terminal or Kelvin techunknown impedance, using ratio arm techniques.

With time, as measurements of more difficult unknown impedances became needed, the original Wheatstone bridge became much modified, with different versions able to measure capacitors or inductors in the series and parallel equivalent mode circuits.

While most Wheatstone bridge circuits suffer from interaction between their resistive and reactive balance controls, certain configurations eliminate this interaction, yielding bridges able to accurately and independently balance either or both terms. A particularly useful publication from Rohde & Schwarz[5], analyses the number of interactive adjustments needed to attain bridge balance, for ten different bridge configurations. One was used to successfully measure the esr of aluminium electrolytic capacitors cooled to –55°C – a most difficult measurement. For more on this topic, see the panel Wheatstone bridges.

What are serial and parallel modes?

The impedance vector of a practical capacitor or inductor at any one given frequency can be represented using an equivalent circuit of the device with a resistor. The resistor, used to degrade the phase angle[1] to that measured, can be either a high value in parallel with the device, or alternatively a low value in series with the device, leading to the term 'equivalent series resistance' or esr.

This esr does not have a finite value. While it is principally frequency dependent, component temperature and applied voltage also have their effects on the measured values.

Have a look at the following practical example. An impedance vector, magnitude 100Ω, phase angle –84.3° at 1kHz, represents a capacitor having a tanδ of 0.1 and Q of 10. This vector would result from a series combination of 9.95Ω resistive and –99.5Ω reactive, i.e. a 1.6μF capacitor or a parallel combination of 1005Ω and 1.584μF. The difference in equivalent capacitance is thus 10% and the equivalent series resistance 9.95Ω.

With practical capacitors, a secondary parasitic, self inductance, will be included in the measured result. Similarly for inductors, a self capacitance value will be included.

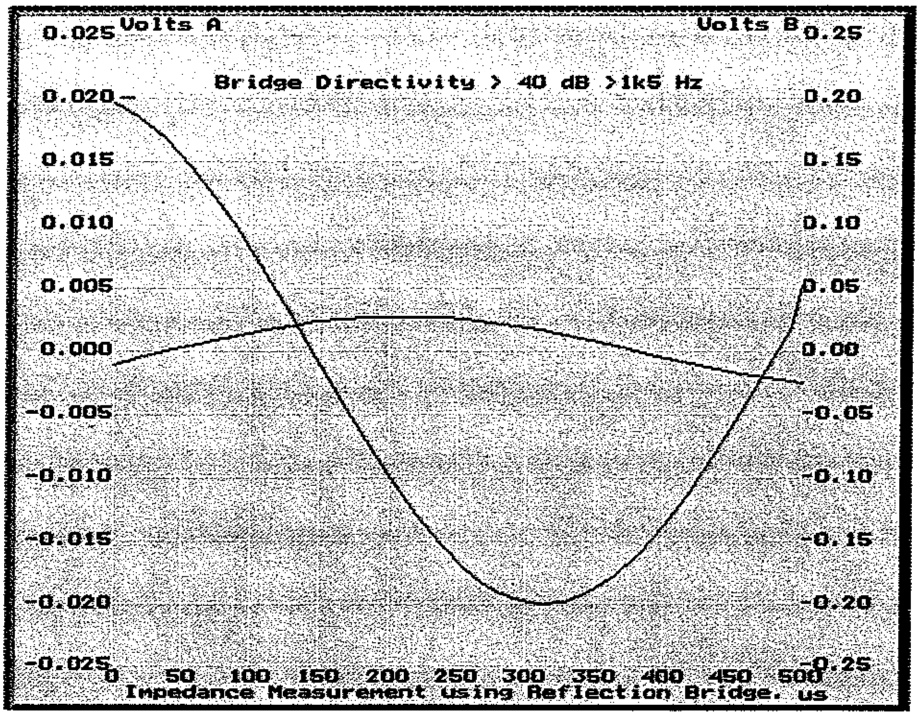


Fig. 1. Using Pico Virtual oscilloscope shows reflection coefficient of bridge loaded with 50Ω. Ideally this reflected voltage would be zero. In practice bridge directivity exceeds 40dB used with this termination at 1500Hz. At higher frequencies directivity improves due to increased impedance of balun used.

Self inductance and self capacitance

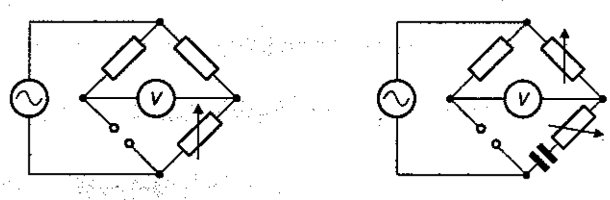
But do these secondary parasitics have any affect on measured values?

You will recall that an inductive reactance has a phase of +90° while a capacitive reactance has a phase of –90°. A simple vector drawing clearly shows that whichever is the minor term, subtracts magnitude from the corresponding major vector component.

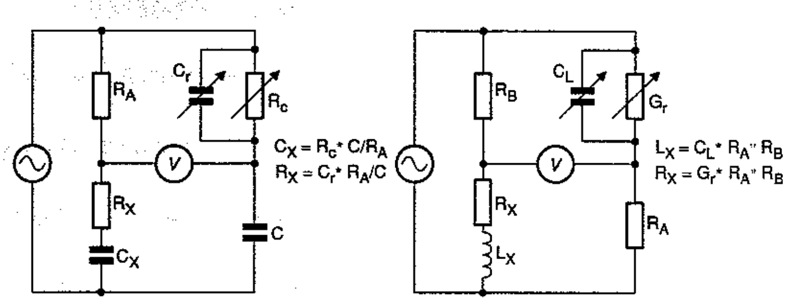
Wheatstone bridges

The conventional Wheatstone bridge used to measure ac impedance of capacitors or inductors, uses a fixed known capacitance standard, together with two calibrated variable resistances. It measures the unknown impedance as a ratio of the known standard, by balancing out the detector voltage to zero. This configuration is used commercially, but suffers from interaction of the two balance controls when measuring low Q or high tanδ components, needs repeated balancing, only slowly approaching true balance.

Other configurations are possible which do not suffer from this interaction and can balance resistive and reactive terms almost independently one from the other. A full analysis[5] of ten optional bridge configurations with estimates of the time to balance, shows improved results when using a variable capacitance to replace one variable resistance. The capacitance bridge shown, built using capacitor and resistor decade boxes, has been used to measure very high esr electrolytic capacitors, when two expensive commercial bridges failed to balance.



AC or dc resistance bridge, left, and ac capacitance bridge.



AC capacitance bridge, left, and ac inductance bridge niques. This ensures that current and voltage are measured using separate pairs of leads. Of course the resistance being measured need not be a discreet component. It might be a track on a printed circuit board, a length of wire or the dc resistance of an inductor.

Four terminals in practice. For low values of resistance, the simplest four terminal measurement is the device-under-test voltage drop when subject to a known constant current. Using a 1A current with a 200mV digital meter, values of a few milliohms can be easily measured. More details are given in the panel entitled four-terminal measurements.

Some years ago, I was faced with the task of measuring resistances from 10mΩ to 100Ω accurately and quickly in an industrial environment. Commercial test meters I looked at were unable to provide a satisfactory solution – regardless of their cost. The components involved could only dissipate a few milliwatts so would not support any significant constant test current. The electrically noisy environment inhibited reliable measurements at the low currents used by commercial meters.

With a test current of less than 1A, power dissipated became excessive at resistance values above 0.1Ω, so a technique that would restrict the power dissipated in the test piece without sacrificing measurement accuracy was needed.

A test current which decreased with increasing device-under-test resistance was arranged using a voltage source and suitable series resistance. Current and voltage drop at the device under test were measured and resistance computed. This resulted in accurate resistance measurements while dissipating only few milliwatts in the test piece.

The final circuit – a trickle charged nickel-cadmium 'D' cell with a 5.6Ω series resistor – provided a short circuit test current less than 250mA, reducing with increasing resistance. This ensured a low, near constant device under test power dissipation. More information is given in the panel entitled Four-terminal measurements.

Four terminals and ac. Alternating-current measurements of low impedance circuits use similar four terminal techniques. They can involve either a known constant current, or a constant voltage source with series resistors.

Voltage, current and phase difference[3] measured using synchronous detectors, provides the basis for many high performance commercial *LCR* test meters. See the panel entitled Phase measurement methods.

While these methods are common in modern automatic *LCR* measuring equipment, they implicitly require meters able to accurately measure currents, voltages and phase angles, at the test frequency.

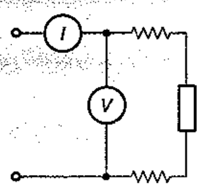
Wheatstone bridge. Long before suitable, easy to use meters were available, a common technique for the dc measurement of resistance, based on ratio arm techniques, was used to compare the unknown, with known resistances.[4] This circuit became known as the Wheatstone bridge.

Subsequently, the Wheatstone bridge was adapted to measure ac characteristics of capacitors and inductors, by adding known capacitor or inductor 'standards' in one arm of the bridge. These were compared with the

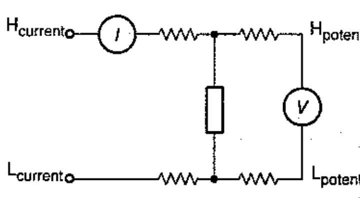
Making four-terminal measurements

Conventional two-terminal measurement of impedance uses the same test lead to both pass the measurement current and measure the voltage drop at the device under test. This results in the measured voltage drop being overstated according to the voltage drop along the test lead used and any contact resistances present.

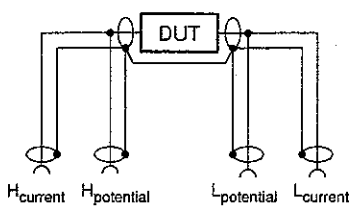
Supplying the test current along one pair of leads while measuring the voltage dropped at the device under test using a second pair of leads – taking care to avoid mutually induced lead voltages – largely overcomes these errors. Use of four coaxial test leads[1] to replace the two pairs of leads, using the screens to carry any return currents together with 'Kelvin' contacts to the device under test, eliminates almost all test lead errors.



Conventional two-terminal circuit for measuring impedance.



Four-terminal circuit.



Measurement configuration.

Occasionally a 'guard' technique, or three-terminal measurement, is used to isolate the device under test from other circuit parasitics. This technique can be upgraded to benefit from the four terminal concepts, when it effectively becomes a six terminal measurement.

The four terminal diagram shown, complete with Kelvin contacts to the device under test, describes the arrangement used for the *HP16047A* component test jig.

Transposing impedance

Series impedances. Any impedance value of the form $|Z|\angle\theta=(|V|/|I|)\angle\theta$ can be transposed from these polar co-ordinates to the rectangular series equivalents of $|Z|=R_s\pm jX_s$ most easily, by using a pocket calculator. Hence also the reverse calculation,

$$|Z|=\sqrt{R_s^2+X_s^2} \qquad \theta=\tan^{-1}\frac{X_s}{R_s}$$

In the above expressions the R term represents the equivalent series resistance of the measured device, while the X term represents the reactive component. When viewed as a vector diagram, a polar plot or on a Smith chart, this X term has a positive value for inductors, and a negative value for capacitors.

With capacitors, the commonly used expressions are,

$$\tan\delta = abs\frac{R_s}{X_s}$$

$$Capacitance=\frac{-1}{\omega X_s}$$

And the equivalent expression for inductors are,

$$Q=\frac{X_s}{R_s}$$

$$Inductance=\frac{X_s}{\omega}$$

Parallel impedances. Certain measuring instruments or mathematical calculations are more suited to the equivalent parallel expression, which can easily be converted to or from the series values,

$$R_p=\frac{R_s^2+X_s^2}{R_s} \qquad X_p=\frac{R_s^2+X_s^2}{X_s}$$

Frequently the measured results are needed as admittance rather than impedance, the conversion from the parallel impedance expression, is simple,

$$Y=\frac{1}{R_p\pm jX_p}=G_p\pm jB_p$$

$$G_p=\frac{R_s}{R_s^2+X_s^2} \qquad B_p=\frac{X_s}{R_s^2+X_s^2}$$

Conversion from parallel impedance to series impedance form is equally simple,

$$R_s=\frac{R_p\times X_p^2}{R_p^2+X_p^2} \qquad X_s=\frac{R_p^2\times X_p}{R_p^2+X_p^2}$$

For more information, see ref 2

ed bandwidths – typically one frequency decade. In practice, they are used only at the higher frequencies. A reflection bridge, made using a carefully wound balun is physically small, and can easily operate over four decades and at low or high frequency.

Traditional Wheatstone bridges are usually manually balanced for zero detector output. But Wheatstone bridges can be used over a limited impedance range without balancing, the resultant error voltage being measured and evaluated.

Designed for use in a 50Ω system and connected to the device under test using a low-loss coaxial cable, such a bridge can measure to high frequencies. Essentially this bridge reduces to three 50Ω precision, non-inductive, resistors together with a wideband 1:1 balun[7].

By changing the balun, this bridge configuration is usable to more than 1GHz or down to audio frequencies. There is more on this in the panel entitled Reflection bridges.

Reflections on transmission lines

Fundamental to a transmission line of any length, terminated by its characteristic impedance, is the complete absorption of all signals travelling towards this termination. Nothing is reflected, **Fig. 1**.

However if terminated other than by this characteristic impedance, regardless of line length[6], a 'reflected' signal is returned back along the line to the source. This signal is called the reflection coefficient since its amplitude cannot exceed that of the signal incident at the line end.

Reflection-coefficient magnitude, ρ, is measured relative to the incident signal and can vary in magnitude from 0 to 1. It is used in the polar notation together with an angle, as $\rho\angle\theta$.

When unterminated, i.e. terminated with an open circuit, the reflected signal's magnitude is identical to that of the incident signal. It is also identical in phase i.e. reflection coefficient is $1\angle 0°$, **Fig. 2**.

When terminated with a short circuit, the reflected signal's magnitude is identical to that of the incident signal, with inverted phase so the reflection coefficient is $1\angle 180°$.

If the termination impedance is resistive and exceeds the line's characteristic impedance, the reflected signal is identical in phase at the end of the line. The signal's magnitude depends on the termination mismatch, **Fig. 3**.

Should this resistive termination impedance be smaller than the line's characteristic impedance, the reflected signal is of opposite polarity, also with a magnitude dependant on the termination mismatch, **Fig. 4**.

Reflection bridges

A reflection bridge, or resistive coupler, resembles the familiar Wheatstone bridge. But while the Wheatstone bridge is normally balanced, the reflection bridge is not adjusted at all. The detector voltage is measured and evaluated instead.

A well designed bridge is usable over four decades of frequency. The detector voltage is described by the vector equation[7],

$$V = 0.125 \times V_o \times \frac{Z_x - Z_o}{Z_x + Z_o}$$

The detector voltage of the conventional Wheatstone bridge is floating and requires a balanced measurement, which is inconvenient at high frequencies. A reflection bridge overcomes this by using a balun transformer to convert detector voltage to an un-balanced output, measurable using conventional meters with coaxial cables.

Most commercial bridges are specified to have at least 40dB directivity over their usable frequency range, i.e. the ability to discriminate to 1% between the incident and reflected signals.

A reflection bridge resolves essentially to three precision resistors together with a precision wound 1:1 balun transformer of primary inductance. This inductance ideally gives 100 times greater impedance, at the required frequency, than the system characteristic impedance. The resistors have a value equal to the required characteristic impedance.

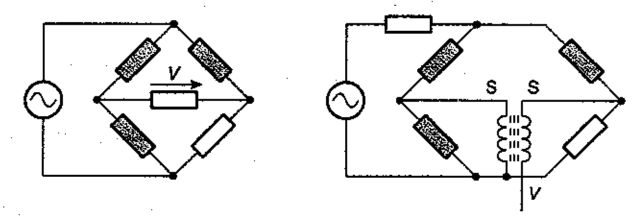
Within these limitations, bridges can be made for use at any desired impedance by amending the resistor values and frequency range. For audio frequencies for example, the balun could be selected for 4, 8 or 50Ω impedance.

The bridge used for the measurements described here was used without applying error correction techniques. It was designed to cover 1kHz to 1MHz frequency range at 50Ω impedance.

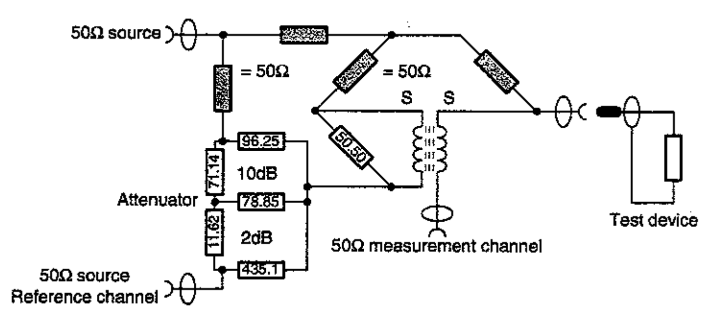
Resistor values used were the same as those of the Hewlett Packard *HP8721A* reflection bridge. The balun was wound with 125 turns of pre-twisted 32SWG wire on a very high permeability ferrite toroid measuring 35 by 22 by 15mm, giving a primary inductance of 240mH. The 2dB attenuator section was adjusted on test, to equalise the reference and measure channel amplitudes during open calibration.

For best accuracy of measurement[8], error reduction techniques can be used to compensate for bridge directivity, source and load matching. This requires the bridge be characterised at the measurement frequency by first measuring known open circuit, short circuit, and 50Ω impedances. These measurements resulting in the vector error factors for effective source match, effective load match and effective directivity, which are used to solve this vector equation, for each measurement and frequency,

$$S_{11a} = \frac{S_{11M} - E_{DF}}{E_{SF} \times (S_{11M} - E_{DF}) + E_{LF}}$$



Wheatstone bridge, left, falls down at higher frequencies but the reflection bridge, right, will produce useful readings at rf.



Reflection bridge configuration used to produce the curves shown in this article.

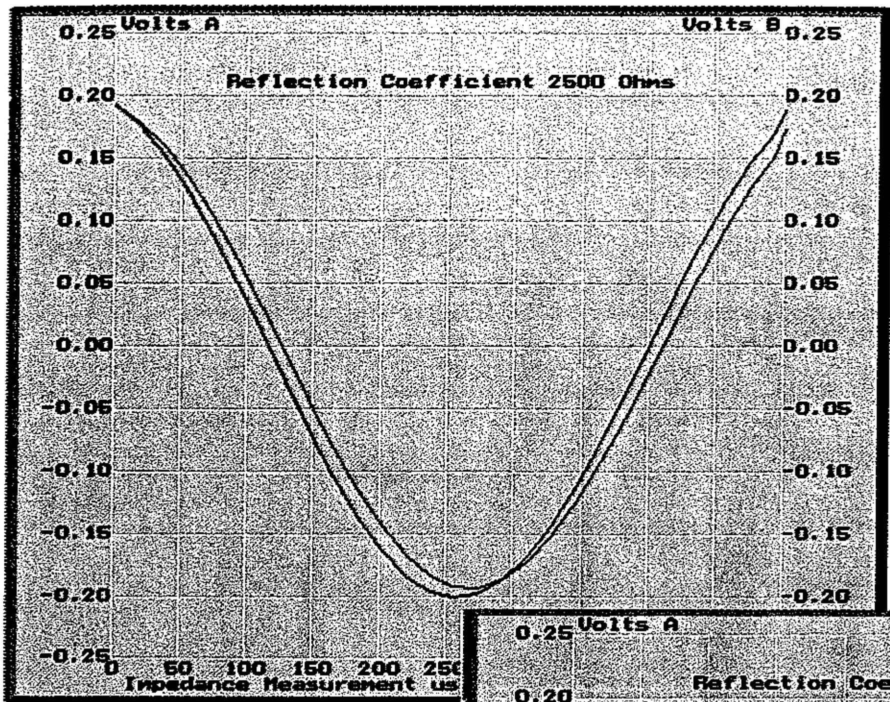


Fig. 2. With the bridge calibrated for equality with open load, this plot shows errors increase as reflection coefficient approaches unity. Measured values – reflection coefficient 0.9586, giving R=2349.8Ω. No error reduction was used. This plot also clearly shows Pico virtual oscilloscope has 10µs delay between channels – channel sampling.

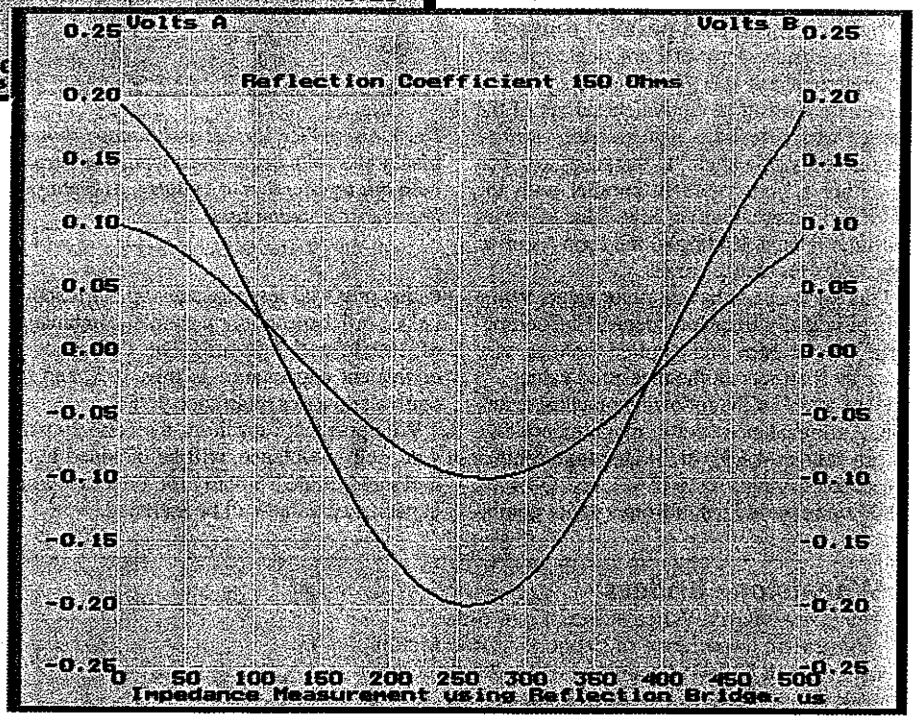


Fig. 3. With 150Ω load, in-phase reflection coefficient is visibly 0.500, giving R=150Ω. Shows this technique can be very accurate with impedances nearer the 50Ω nominal Z_0. Only open calibration used – no error reduction.

A smaller vector magnitude[1] represents a reduced capacitive reactance. When inductance is added in series with a capacitor, the equivalent or measured capacitance is increased. Conversely self capacitance added in parallel with an inductor decreases its measured or equivalent values. Consequently both capacitance and inductance measured values are frequency dependent.

These impedance measurement methods can be used from audio frequencies to a few megahertz. Reliability of readings at the higher end depends on the parasitics involved in connecting the measurement meters to the test device and the phase meter's upper frequency limit.

At low frequencies, the traditional Wheatstone bridge measurement performs extremely well, but due to internal parasitics in the bridge, is usually restricted to less than 1MHz. Additionally at 1MHz, 1m of test coaxial cable gives a phase shift of 1.8°. Since this cannot be fully compensated, makes tanδ or Q measurements of low-loss parts at 1MHz, extremely difficult.

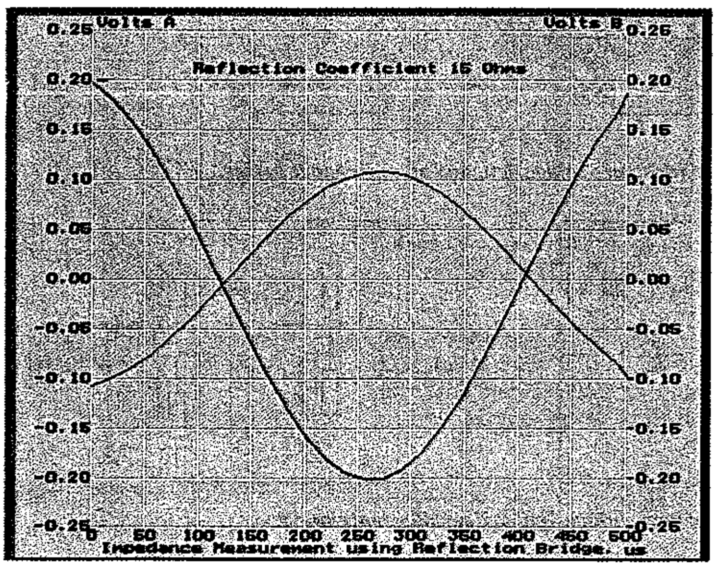


Figure 4. With impedances less than Z_0 the reflection coefficient is clearly phase reversed. Measured value of reflection coefficient = –0.54166 giving R=14.865Ω. All test resistors from Muirhead precision decade resistor box. Only open calibration used – no error reduction.

And measurements above 1MHz?

Impedance measurements at higher frequency require either expensive dedicated measurement systems, or the use of the vector network analyser techniques.

A vector network analyser overcomes the problem of test device connection by measurement using either a reflection bridge[6] or directional coupler. This is connected to the test device using low loss coaxial cable. Intermediate frequency superhet methods are used to down convert the test frequency, to overcome phase meter frequency limits

Directional couplers must be electrically long and hence can operate only over limit-

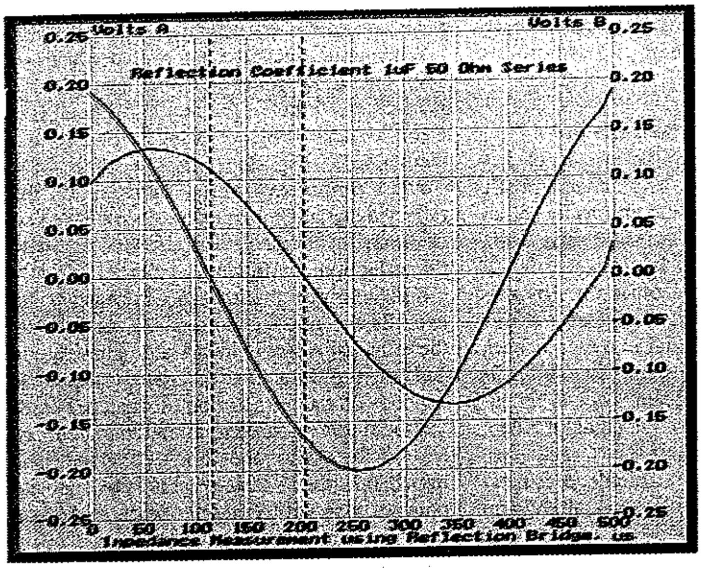


Fig. 5. Worked example 1, for 1μF polypropylene capacitor in series with 50Ω.
Reflection coefficient is 0.6665∠–48.919.
Polar to rectangular conversion Γx=0.43797, Γy=–0.502395.
Solve for *R*±jX, *R*=48.9Ω, *X*=–88.406Ω.
Solving for component value at test frequency of 1786Hz, equivalent series resistance=48.9Ω and capacitance=1.008μF

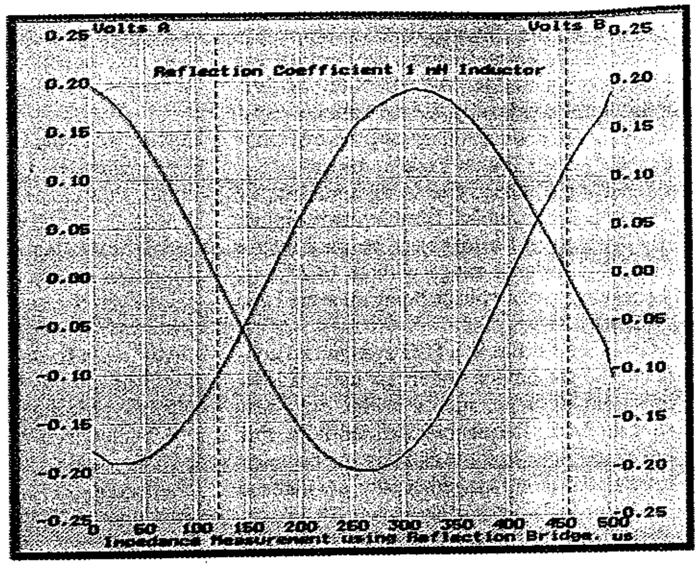


Fig. 6. Worked example 2, for a 1mH ferrite-core inductor with no added resistance.
Reflection coefficient is 0.955∠–205.087
Polar to rectangular conversion Γx=–0.86491, Γy=0.40491
Solve for R±jX, R=1.208Ω, *X*=11.118Ω
Solving for component value at test frequency of 1786Hz, equivalent series resistance=1.208Ω and inductance=0.9908mH

Terminated with a pure capacitive or inductive load, the reflected phase is load dependent, and the magnitude equals the incident wave. Now the reflection coefficient is 1∠θ.

Phase angles calculated from time difference measurements, as in these examples, are read as being of negative phase, compared with the reference signal. Reflection Coefficient can be readily plotted on a Smith chart. But since the Smith chart uses negative angles to represent capacitive reflection coefficients and positive angles to represent inductive reflection coefficients, measured angles exceeding –180° should be normalised by adding +360°. Once plotted on a Smith chart[6], converted *R*±j*X* values are immediately available.

In rectangular notation the reflection coefficient is given the symbol Γ, having both a magnitude and phase value in the form, ±Γx±Γy.

Reflections to component values

In rectangular notation, the reflection coefficient can be converted into conventional component parameters of *R*±j*X*, simply by substitution in two standard equations. If measured using polar notation, the reflection coefficient must first be translated into rectangular notation[2], using a pocket calculator P→R function,

$$R = Z_0 \times \frac{1-(\Gamma x^2 + \Gamma y^2)}{1 - 2\Gamma x + \Gamma x^2 + \Gamma y^2}$$

$$jX = Z_0 \times \frac{2\Gamma y}{1 - 2\Gamma x + \Gamma x^2 + \Gamma y^2}$$

Having converted the measured reflection coefficient into *R*±j*X* format, any other desired impedance transpositions are simply performed. For more on this, see the panel Impedance transpositions and **Figs 5, 6**.

Measurement accuracy. At these frequencies, output from the test generator may be maintained constant. However, having passed through connecting cables and the reflection bridge, the test signal amplitude will suffer from variations, especially with test device loading.

These amplitude variations are compensated by performing all measurements relative to a portion of the generator output tapped off, using a two way splitter, for measurement by the reference channel.

Connecting this bridge to the test device at high frequencies is made easy by its inherent ability to discriminate between two signals travelling in opposite directions. Consequently the cable used to supply the test stimulus to the device under test also conveys the reflected error signal which is measured.

The whole system, including connecting cables, is calibrated using known open circuit, short circuit and matched 50Ω termination. These must be connected at the exact point where the device-under-test is to be connected – known as the reference plane – using the frequencies to be measured.

Accuracy enhancement. Obviously, this reflection bridge has limited discrimination and isolation of the forward and reflected signals. Typically, its directivity exceeds 40dB, making it better than 100:1.

The calibration routines using known open, short and 50Ω loads also allow the system errors to be measured. As a result, mathematical error reduction techniques can be used to correct for these deficiencies[8]. There is more on this in the panel Reflection bridges.

In addition simple techniques also exist to measure then correct for[2] and remove errors introduced by any test jig used to house the test piece.

Measuring in practice

Using these techniques together with the Hewlett Packard *8753* vector network analyser and coaxial test jig *HP16091A*, I have made many measurements of surface-mount miniature high-Q ceramic chip capacitors. These measurements have been made from 1MHz to 3GHz, with excellent repeatability between results.

Such high frequency measurements complement the lower frequency measurements, made using precision *LCR* meters. ■

References

1. Botos, R, 'Designer's Guide to RCL Measurements'. Hewlett Packard pub. No 5952-4800
2. Using the 4191A RF Impedance Analyser, Hewlett Packard app note 302-2.
3. Optimising Electronic Component Impedance Measurements, Hewlett Packard App. Note 369-1.
4 Reference Data for Radio Engineers, Butterworth Heinemann.
5. Precise Balancing of Bridges, News from Rohde & Schwarz 23
6. Vector Measurements of High Frequency Networks. Hewlett Packard pub No 5954-8355.
7. Broadband Impedance Measurements – Bridge ZRB2, News from Rohde & Schwarz106.
8. Fitzpatrick, J, 'Error models for systems measurement', *Microwave Journal* May 1978.