Multilayer air-cored coils

If you want to calculate the inductance of a coil and you know all the coil's other specifications, Wheeler's equation provides a simple and accurate solution. But the designer's task is usually the reverse. Robert Kesler describes the return path.

The most practical way to calculate the inductance of an air-cored multilayer coil is via Wheeler's formula.[1]

$$L = \frac{7.87N^2M^2}{3M+9B+10C} \quad (1)$$

This formula is claimed to be accurate to within 1%, if the numbers in the denominator are about equal, that is, if the shape of the coil is similar to that shown in the diagram. Of course, the accuracy in practice is also determined by the tolerances on the coil's dimensions.

A given piece of wire, wound in a coil, yields the highest inductance value, if its proportions are, $3M=9B=10C$. **It can be proved** that this shape yields the highest inductance for a coil, wound from a given piece of wire, or the highest inductance/resistance ratio for a given weight of copper.

The Wheeler formula is fine for calculating the inductance of a coil when everything else is known. But usually, the designer's task is the opposite: at the outset, only the inductance is known. In addition to the four other parameters in the formula, the wire diameter has to be found too.

Working back

A coil's dc resistance can be expressed using the diameter and the length of the copper wire, which can be expressed using the number of turns, and the mean diameter of the coil,

$$R = \frac{NM}{14250W^2} \quad (2)$$

The relationship between the number of turns that can be packed into the cross section of the coil and the wire diameter is,

$$N\left(\frac{W}{P}\right)^2 = BC \quad (3)$$

Considering only the ideal shape, there are two more useful equations,

$$B_i = \frac{M_i}{3} \quad (4)$$

$$C_i = 0.3M_i \quad (5)$$

In this case, formulas 1 and 3 become simpler,

$$L = 0.875N_i^2M_i \quad (6)$$

$$N\left(\frac{W}{P}\right)^2 = 0.1M_i^2 \quad (7)$$

Now the parameters can be expressed, using the formulas 2, 6 and 7,

$$M_i = 0.354\sqrt{\frac{L}{R}} \quad (8)$$

$$N_i = 1.07\sqrt{\frac{L}{M_i}} \quad (9)$$

$$W_i = 0.253\frac{M_i}{\sqrt{N_i}} \quad (10)$$

the inner and the outer diameter of the coil,

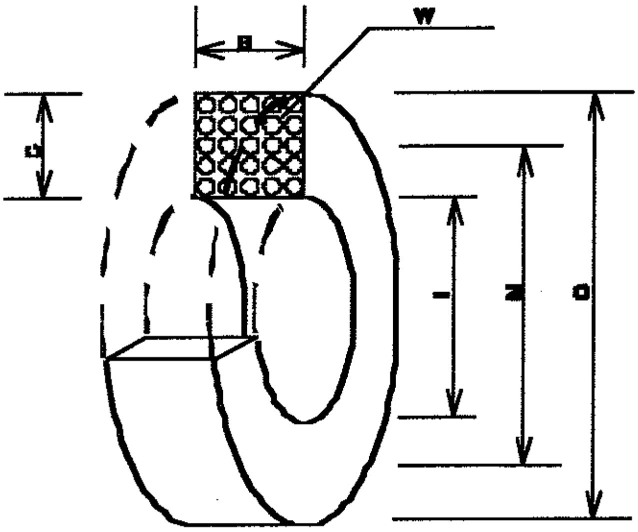
$$I = M - C = 0.7M \quad (11)$$

$$O = M + C = 1.3M \quad (12)$$

and two more useful relationships,

$$M_i = 3.08L^{0.2}W^{0.8} \quad (13)$$

$$N_i = 0.61\left(\frac{L}{W}\right)^{0.4} \quad (14)$$



L inductance in nanohenries
R resistance in ohms
N number of turns
M mean diameter
B width or length
C radial thickness
I inner diameter
O outer diameter
W diameter of the copper wire
P linear packing density*

(the wire diameter divided with the centre-to-centre wire spacing)
all dimensions in mm.

Air-cored coil features.

Note that formulas containing 'i' subscripts are valid only for ideally shaped coils.

Tips for designing multilayer coils

The above formulas are useful tools for designing multilayer air-cored coils. Two starting points are possible. With the first, the dc resistance is initially specified. For the second, dimensions of an available coil former need to be known.

If the coil is to be used in a high power circuit such as a loudspeaker crossover network, start by specifying the maximum allowable losses in terms of the dc resistance. Calculate the theoretical wire diameter using formulas 8, 9 and 10. Choose the nearest available standard wire size.

The calculated diameter need not be adhered to rigidly, bearing in mind that increasing the diameter will decrease the resistance, but increase the dimensions, and *vice versa.*

Use the chosen wire diameter to calculate the dimensions and the number of turns with formulas 1, 4, 5, 11, 12 and 14.

When working out from a given coil former, calculate the number of turns and the wire diameter, using formulas 1 and 3, with the dimensions of the former. Choose the nearest standard wire size. Using these new dimensions, recalculate the radial thickness and the mean diameter of the coil, then the number of the turns.

Calculate the dc resistance and decide whether or not it is acceptable. Note that in all the calculations, the linear packing density is assumed to be 0.8. Any errors that may arise from this assumption and other causes can be corrected at the final stage.

Measure the diameter of the ready wound coil. Before cutting the wire, calculate the inductance using formula 1 and correct the number of turns, as necessary.

Design examples

This first example shows how to proceed when the initial known quantity is the dc resistance of the coil. The inductance required is 200μH and the resistance assumed is 0.3Ω.

For mean diameter, number of turns and wire diameter,

$$M_i = 0.354\sqrt{\frac{200.10^3}{0.3}} = 28.9mm \qquad (8)$$

$$N_i = 1.07\sqrt{\frac{200.10^3}{28.9}} = 89 \qquad (9)$$

$$W_i = 0.253\frac{28.9}{\sqrt{89}} = 0.775mm \qquad (10)$$

At this point, the calculated wire diameter can be modified to suit available sizes, provided that you understand the consequences.

Let the new wire diameter be 1mm. Use this diameter in the subsequent calculations,

$$M_i = 3.08\times 200000^{0.2}\times 1^{0.8} = 35.4 \qquad (13)$$

$$B_i = \frac{35.4}{3} = 11.8 \qquad (4)$$

$$C_i = 0.3\times 35.4 = 10.6 \qquad (5)$$

$$I = 35.4 - 10.6 = 24.8mm \qquad (11)$$

$$O = 35.4 + 10.6 = 46mm \qquad (12)$$

$$N_i = 0.61\left(\frac{200000}{1}\right)^{0.4} = 132 \qquad (14)$$

Having finished the winding, remember to measure the outer diameter of the coil before cutting the wire. Calculate the inductance from formula 1, and correct the number of turns, if necessary.

The second design example is for when you are working from a specified coil former. Inductance required is 2mH. Assume that the former has an inner diameter, *I*, of 12mm, a width, B of 16mm and a rim, *C*, of 6mm.

$$2\times 10^6 = \frac{7.87\times N^2\times 18}{3.18+9.16+10.6} \qquad (1)$$

N=450

$$450\left(\frac{W}{0.8}\right)^2 = 18.6 \qquad (3)$$

=0.392mm.

Choose the next available smaller wire diameter, *W*, of 0.35mm then calculate the radial thickness of the coil,

$$450\left(\frac{0.35}{0.8}\right)^2 = 16C \qquad (3)$$

C=5.4 mm.

The new value for the radial thickness – and the mean diameter – may call for re-calculation of the inductance and correction for the number of turns. Alternatively, it may be left to the final correction, before cutting the wire.

Check the dc resistance,

$$R = \frac{17.6\times 450}{14250\times 0.35^2} = 4.54\Omega \qquad (2)$$

Decide whether this value is acceptable. If too high, choose a former of bigger diameter and/or cross section. ■

Further reading

H. A. Wheeler, Simple Inductance Formulas for Radio Coils, *Proc. IRE,* Vol. 16, p. 1398, Oct. 1928,

F. E. Terman, Radio Engineer's Handbook, McGraw-Hill, 1943. p. 62.