Grounding on a different plane

Like any conductor carrying current, a ground plane has voltages across it and these voltages can cause emc problems. Ian Darney explains how to analyse and solve such problems.

The 'equipotential ground plane' is a concept often used by system designers. As an aid to the practical problems of controlling interference, however, it owes more to mythology than science; its usefulness is about the same as a talisman brandished to ward off evil spirits.

The purpose of this article is to prove the validity of the last statement, and, in so doing, to introduce a method of analysing interference in electronic systems – a method that cannot be found in any book.[1]

Essentially, this method is a procedure for deriving the capacitive and inductive parameters of wiring assemblies, and creating circuit models.

The formulation is introduced in terms of the coupling between two wires over a ground plane, but can be developed to cater for more complex configurations. Even so, it is not complicated.

Anyone whose eyes have glazed over at the mention of div, curl, del, Maxwell equations, or boundary conditions, can relax. Most of the mathematical operations involved in the formulation are simple addition and subtraction.

Method of analysis

Traditionally, the effect of the ground plane is simulated by image conductors, and this treatment is no exception. For two wires over a plane, the number of conductors to consider must be four; two real and two image conductors.

A set of four 'primitive' equations is defined, relating voltages on the wires to the currents they carry. When this set of conductors is configured to carry signals, two loops are involved. This leads to two 'loop' equations, which can be replicated by two 'circuit' equations, derived from a circuit model.

'Primitive' parameters relate to the performance of the conductors as antennae, 'loop' parameters can be measured using test equipment, and 'circuit' parameters are those found in a circuit diagram. This distinction is important.

A circuit model is created to generate two mesh equations. Components of the circuit model are then related to the inductive, capacitive and resistive elements of the primitive equations. Acknowledgment of the fact that current varies along the length of any signal carrying conductor leads to the e of T-networks in the final circuit model.

A simple example is used to illustrate the response of a 'victim' circuit in one conductor/ground-loop to the presence of a step voltage in the other loop. No tedious calculations are involved, since mathematical and circuit analysis software is used to full advantage.

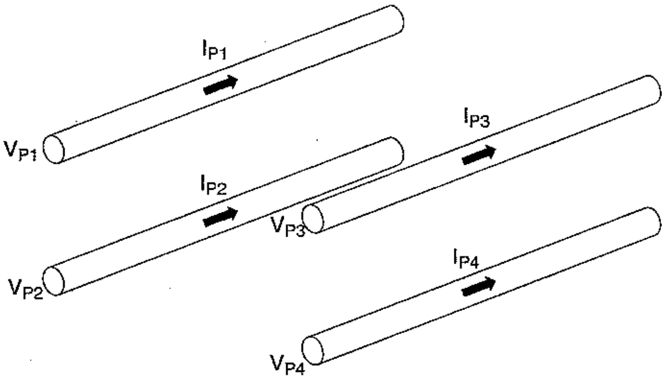


Fig. 1. Primitive – i.e. absolute – voltages and currents in four conductors

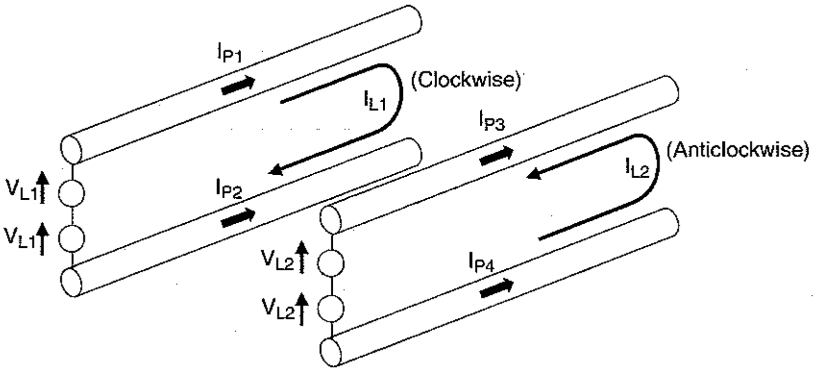


Fig. 2. Loop voltages and currents involved when two voltage sources are applied – one to the ends of conductors 1 and 2, the other to conductors 3 and 4.

This first example demonstrates conclusively that there is no such thing as an equipotential ground plane in a functioning system.

Shielding of any signal conductor is achieved, simply by routeing a second conductor along its length, and grounding this second conductor at both ends. Characteristics of the

Formulations

Consider the four isolated wires of **Fig. 1**. The configuration can be visualised as a section of a long structure, allowing end effects to be ignored. Assuming sinusoidal voltages and currents exist in the conductors, then,[2]

$$\begin{aligned} V_{P1} &= Z_{P11}\times i_{P1} + Z_{P12}\times i_{P2} + Z_{P13}\times i_{P3} + Z_{P14}\times i_{P4} \\ V_{P2} &= Z_{P21}\times i_{P1} + Z_{P22}\times i_{P2} + Z_{P23}\times i_{P3} + Z_{P24}\times i_{P4} \\ V_{P3} &= Z_{P31}\times i_{P1} + Z_{P32}\times i_{P2} + Z_{P33}\times i_{P3} + Z_{P34}\times i_{P4} \\ V_{P4} &= Z_{P41}\times i_{P1} + Z_{P42}\times i_{P2} + Z_{P43}\times i_{P3} + Z_{P44}\times i_{P4} \end{aligned} \tag{1}$$

where the subscript '*P*' identifies all primitives. If the integers *i* and *j* identify individual conductors, then the primitive impedances can be defined,

$$Z_{Pij} = j\omega L_{Pij} + R_{Pij} + \frac{1}{j\omega C_{Pij}} \tag{2}$$

$$L_{Pij} = \frac{\mu l}{2\pi}\left(\ln\left(\frac{2l}{r_{ij}}\right) - 1\right) \tag{3}$$

$$C_{Pij} = \frac{2\pi\varepsilon l}{\ln\left(\frac{l}{r_{ij}}\right)} \tag{4}$$

$$\begin{aligned} R_{Pij} &= \text{conductor resistance } i \\ &(R_{Pij} = 0 \text{ if } i \neq j) \end{aligned} \tag{5}$$

where *j* is the complex operator, ω is the angular frequency, r_{ij} is the separation between the axes of conductors *i* and *j*, r_{ii} is the radius of conductor *i*, *l* is the length, while ε and μ are the permittivity and permeability of the insulation.

Assumptions inherent in equations (3) and (4) are that $l >> r_{ij}$, that the charge and current are evenly distributed on the surface of each wire, and that the concept of 'action at a distance' is valid.

The formulae for primitive inductors (3) and capacitors (4) may seem unusual, because these parameters are used to describe the characteristics of the conductors when the assembly is acting like an antenna. They relate the surrounding magnetic and electric field to current in the structure. Quantities defined in terms of henries or farads can have more than one interpretation!

One feature of primitive impedances is that they are symmetrical. Since $r_{ij}=r_{ji}$, it follows that $Z_{Pij}=Z_{Pji}$. Now assume that a voltage source, $2V_{L1}$, is applied between the ends of conductors 1 and 2, and that a voltage source, $2V_{L2}$, is applied between the corresponding ends of conductors 3 and 4, as shown on **Fig. 2**. Loop voltages with subscript '*L*' can be related to primitive voltages,

$$\begin{aligned} 2V_{L1} &= V_{P1} - V_{P2} \\ 2V_{L2} &= V_{P3} - V_{P4} \end{aligned} \tag{6}$$

Relationships between loop currents and primitive currents are defined in Fig. 2.

$$\begin{aligned} i_{L1} &= i_{P1} = -i_{P2} \\ i_{L2} &= -i_{P3} = i_{P4} \end{aligned} \tag{7}$$

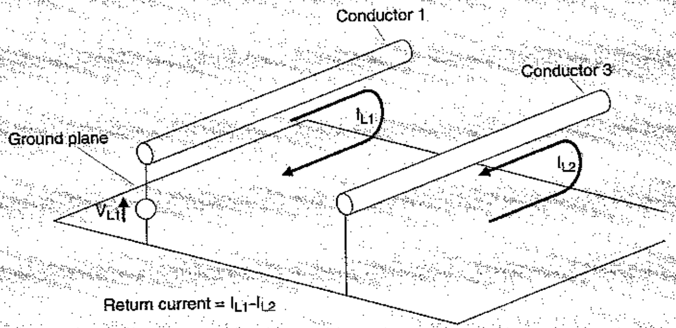


Fig. 3. Here. conductors 2 and 4 are replaced by a ground plane.

If conductors 2 and 4 are replaced by a ground plane and V_{L2} is replaced by a short circuit, the picture changes to that shown on Fig. 3. The relationships of equations (6) and (7) continue to apply. Substituting them in equation set (1) and taking account of symmetry of the image conductors leads to the loop equations,

$$\begin{aligned} V_{L1} &= Z_{L11}\times i_{L1} + Z_{L12}\times i_{L2} \\ 0 &= Z_{L21}\times i_{L1} + Z_{L22}\times i_{L2} \end{aligned} \tag{8}$$

where,

$$\begin{aligned} Z_{L11} &= Z_{P11} - Z_{P12} \\ Z_{L12} &= Z_{P14} - Z_{P13} = Z_{L21} \\ Z_{L22} &= Z_{P33} - Z_{P34} \end{aligned} \tag{9}$$

The main characteristic of loop parameters is that they can be measured with test equipment. Another characteristic is that they are symmetrical; $Z_{L12}=Z_{L21}$.

So far, the formulation has been in terms of electromagnetic theory. To create a circuit model, it is necessary to indulge in an exercise of lateral thinking and consider the question; "what configuration of components could be used to mimic the relationship between loop voltages and loop currents of equation (8)?"

Given the problem in these terms, it does not take long to sketch out **Fig. 4**, where the subscript 'C' identifies circuit impedances. Comparing this with the equivalent circuit of a magnetic transformer identifies an earlier application of the concept.

Mesh analysis of Fig. 4 gives rise to the circuit equations,

$$\begin{aligned} V_{L1} &= (Z_{C11} + Z_{C12})\times i_{L1} - Z_{C12}\times i_{L2} \\ 0 &= -Z_{C12}\times i_{L1} + (Z_{C12} + Z_{C22})\times i_{L2} \end{aligned} \tag{10}$$

Correlating the terms of the circuit equations (10) with those of the loop equations (8) allows each circuit impedance to be defined in terms of loop impedance. Using equations (9), the circuit impedances are then related to the primitives.

$$\begin{aligned} Z_{C12} &= -Z_{L12} = Z_{P13} - Z_{P14} \\ Z_{C11} &= Z_{L11} + Z_{L12} = Z_{P11} - Z_{P12} - Z_{P13} + Z_{P14} \\ Z_{C22} &= Z_{L12} + Z_{L22} = Z_{P33} - Z_{P34} - Z_{P13} + Z_{P14} \end{aligned} \tag{11}$$

Since each primitive impedance is defined in terms of inductance, capacitance, and resistance in equation (2), each circuit impedance will also contain *L*, *C* and *R* parameters. Using equations (3), (4), and (5), the formula for each component can be derived. For resistors, the derivation is simple,

$$\begin{aligned} R_{C11} &= R_{P11} \\ R_{C12} &= 0 \\ R_{C22} &= R_{P33} \end{aligned} \tag{12}$$

It is possible to write out the formulae for reactive components merely by inspecting equation set (11). These are listed in the Appendix, under the heading "derivation of electrical parameters".

Detailing the elements of Fig. 4 leads to the initial circuit model, **Fig. 5**. This is rather crude, since it overcompensates for the fact that current in the capacitors will alter the voltages in the inductors. A better model can be formed, using the familiar T-junction. This leads to **Fig. 6**.

By invoking the star-to-delta transformation shown in the Appendix, the node at the junction of the three capacitors can be removed, giving the final circuit model of **Fig. 7**.

Using equation set (12), the resistors of Fig. 7 can be related to hardware.

With a one-to-one correlation between the conductors of Fig. 3 and the horizontal branches of the circuit, Fig. 7 can be treated as a section of a three-conductor transmission line. The simulation is surprisingly accurate[2].

new configuration are assessed without difficulty.

Using this technique with modern software, the design of equipment can be tailored to meet emc requirements.

Coupling via the ground plane

To put flesh on the bones of the dry theory in the panel entitled 'Formulations', the first action is to assign dimensions to the physical parameters of **Fig. 3** shown in the panel. One example is shown in **Fig. 8**.

At this point the value of mathematical software[3] becomes evident. When such software is accessed, the relevant equations can be typed on to the computer screen as easily as words when a word processor is used. The Appendix presents the entire calculation process, including the input data and the tabulation of results. Plugging these values into Fig. 7 gives the basic structure of **Fig. 9**.

In this example, conductor 1 is defined as the 'culprit', and fed with a 5 V step voltage, V_{in}, via a 100Ω resistor. The load at the far end is 1kΩ. Conductor 3 is nominated as the 'victim' conductor.

The victim circuit is shorted at one end and loaded with Ω at the other. Voltage V_{out} across the resistor is deemed to be the signal under review. No conductor has zero resistance, so a finite value has been assigned to the ground plane.

Having created a circuit model, the next step is to analyse it, and this brings into play the processing power of circuit analysis software[4]. In this application, such software is even more impressive than the mathematical variety, because it obviates the need to check any equation.

All that is necessary is to draw the circuit on the screen and define component values. The computer program generates the necessary vectors for use in the analysis. The type of analysis, the test limits, and the signals to be examined are then selected, and at the touch of a key the output is presented on the screen.

For the circuit of Fig. 9, the transient voltage V_{out} is shown in **Fig. 10**. This should be no surprise to anyone who has monitored logic signals on an oscilloscope using a voltage probe. The spurious signal has the classical damped sinusoidal waveform, indicating an oscillatory condition.

Reflections

need to puzzle very long to identify the reason for this: e loads at each end of the transmission line are causing reflections.

Charges are surging backwards and forward along the two metre line, mostly along the ground conductor. Energy is stored dynamically in the electric and magnetic field surrounding the conductors.

With the ground plane acting as an inductor and oscillatory current flowing in that inductor, a significant voltage is developed along the length. Since the victim loop utilises the ground plane to form its return conductor, this voltage appears in series with the expected signal in that loop.

A spurious signal, V_{out} is received by any circuit which monitors the voltage across R_{victim}. From the point of view of the victim circuit, current in the structure is emanating from an external source. Circuit theory tells us that the same effect can be achieved by a voltage generator in series with the ground conductor. The ground plane acts as a voltage source. It is anything but an equipotential surface.

However that is not the only deduction. In addition to predicting the existence of a ringing signal, Fig. 10 provides information on the frequency of oscillation (about 33MHz) and on the amplitude, peaking at about 200mV. This par- ilar resonance correlates with the quarter wave frequency. Further development of the circuit model to include the

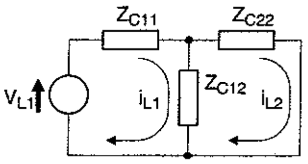


Fig. 4. This circuit gives a set of equations similar to loop equations.

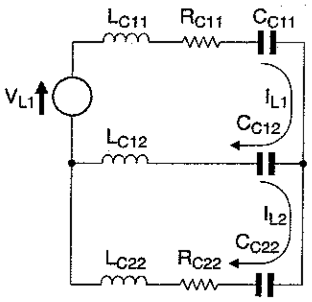


Fig. 5. Initial circuit model, identifying individual components.

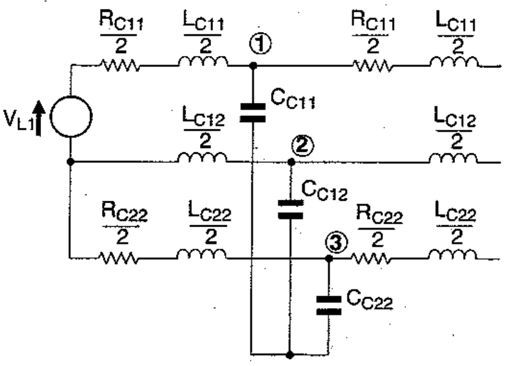
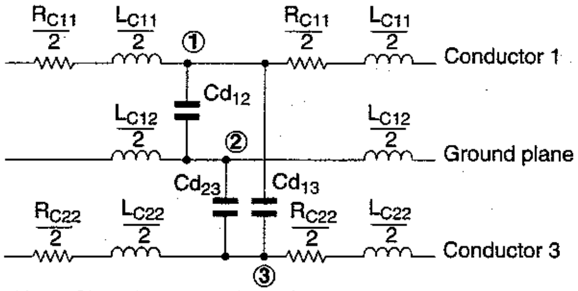


Fig. 6. The model is improved by converting each branch to a T network.



Note: Circled numbers identify nodes common to Figs 6 and 7

Fig. 7. Final circuit model after transformation from star to delta.

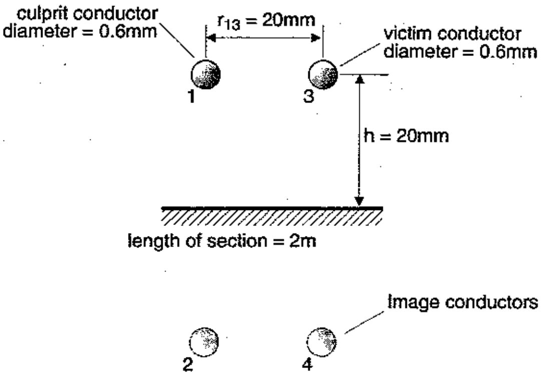


Fig. 8. Physical dimensions of the two wires over a ground plane used in the example.

active components which interface with the conductors is a simple matter. If the signal across R_{victim} is to be monitored by a logic gate, then simulation will show whether or not the device will generate a spurious pulse in response. Measures can be taken, at the initial design stage, to prevent such an occurrence.

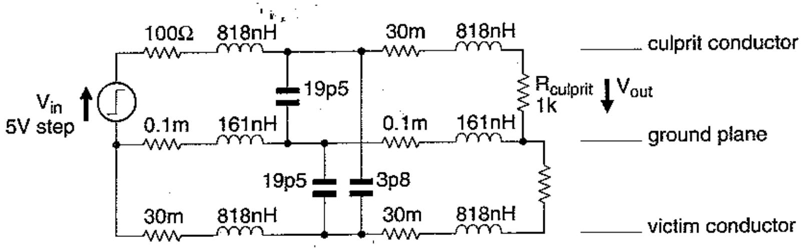


Fig. 9. Circuit model of two wires over a ground plane.

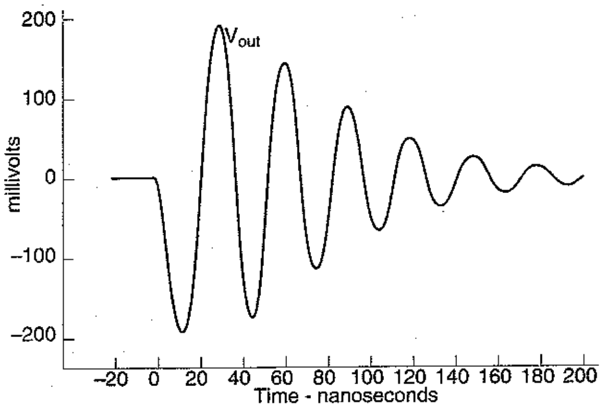


Fig. 10. Response of victim circuit to a 5V step in culprit circuit.

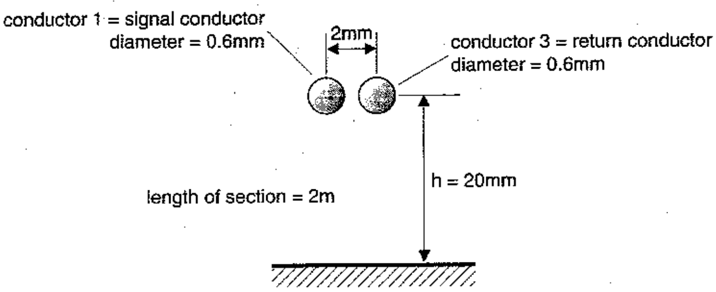


Fig. 11. One way to reduce interference is to allocate a second conductor to carry the return current. This should be mounted as close as possible to the one carrying signal current.

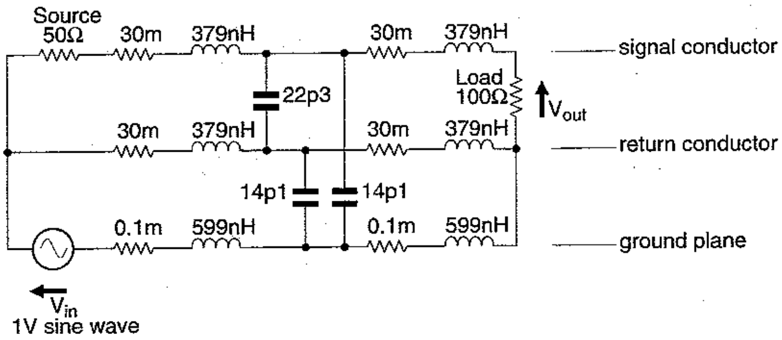


Fig. 12. Circuit model for the two wires over a ground plane configuration shown of Fig. 11.

Shielding: the first step

Perhaps the most important way of reducing interference is to allocate a conductor to carry return current, and to route that conductor as close as possible to the one carrying signal current. **Figure 11** shows the new wiring configuration.

Exploring the implications of this is quite a simple task. Going to the Appendix and altering the value of r_{13} from 20mm to 2mm will change all the reactive values in the tabulation of results.

The simplest way of ensuring that conductor 3 acts as a return conductor is to link it to local structure – the ground plane – at both ends. Values of 50Ω and 100Ω are fairly representative of the source and load resistors. **Figure 12** illustrates the effect of these changes to the model.

Inspection of this circuit reveals that the inductance of the return conductor is less than that of the ground plane, even though it presents a relatively high resistance. So this conductor is indeed the preferred return path for hf signal current.

Capacitance between signal and return conductors will enhance this effect. As the frequency increases, the performance of the wire pair approaches that of the ideal transmission line, the most efficient way of carrying electrical energy from source to load.

To determine the effect of interference, **Fig. 12** includes a sinusoidal voltage source of 1V in series with the ground. Carrying out a frequency response analysis, again using computer aided design,[4] results in **Fig. 13**. This shows that between 10kHz and 10MHz, the attenuation of any spurious signal is more than 10dB.

Above 10MHz capacitive effects cause the attenuation to increase further, but beware of resonance peaks! The model predicts one at 43MHz, pointing towards the half wave resonance. The dip in the curve correlates with quarter wave resonance. Actual tests would determine the frequency of the dip and peak more accurately.

Even in the presence of resonance, a wire pair will reduce interference.

Implications

Treating the structure as a large diameter conductor results in three primitive equations and two loop equations. Circuit model Fig. 7 remains valid, even though component formulae change. By increasing the number of conductors in the configuration, more complex circuit models can be created, allowing all the usual circuit interconnections to be analysed.[2]

Using general-purpose test equipment such as the oscillo-

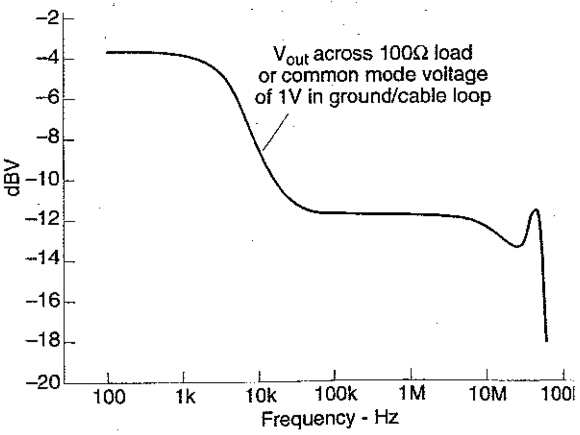


Fig. 13. Differential-mode response of wire pair over a ground plane.

scope, signal generator and toroidal transformer, it is possible to make practical measurements and build up circuit models of actual hardware[2]. Circuit analysis of the models will allow system performance during formal emc testing to be predicted.

Transient analysis and frequency analysis of complex circuits are simple tasks with modern software[4]. Anyone who can design electrical equipment can learn to analyse emc.

In summary

Two conclusions of practical importance are,

- It is a bad mistake to assume that the ground plane is an equipotential surface, or that two points marked with the earth symbol are at the same voltage.
- A significant improvement in emc can be realised, simply by allocating two conductors to every signal and routeing those conductors together.

Of even greater importance is the fact that circuit modelling can be used to assess the merits and failings of any particular system, allowing design decisions to be based on analysis rather than on a debatable set of guidelines. ■

References

1. Williams, Tim, EMC for Product Designers, Appendix B:CAD for EMC, pub. Newnes, 1996.
2. Darney, Ian, 'Circuit Modelling for Electromagnetic Compatibility,' *Electronics & Communications Engineering Journal*, pp. 184-192, August 1997.
3. Mathcad plus 6.0 (Adept Scientific plc, 6 Business Centre West, Avenue One, Letchworth, Hertfordshire SG6 2HB).
4. 'Geseca for Windows: Spiceage for Windows' (Those Engineers Ltd, Mill Hill, London NW7 4BP).

Appendix – coupling calculations.

These are calculations for coupling between two wires over a ground plane. Mathcad was used to compute the results.

$$\mu := 4\pi 10^{-7}\,H/m \quad \varepsilon := 8.854\times10^{-12} \quad \rho := 1.7\times10^{-8}\,\text{ohm.m (resistivity of copper)}$$

$$\mathrm{r}_{1,1} := 0.3\times10^{-3} \quad \mathrm{r}_{3,3} := 0.3\times10^{-3} \quad \mathrm{r}_{1,3} := 20\times10^{-3} \quad \mathrm{h} := 20\times10^{-3} \quad l := 2\mathrm{m}$$

Derived dimensions

$$\mathrm{r}_{1,2} := 2\times h \quad \mathrm{r}_{1,4} := \sqrt{(r_{1,2})^2 + (r_{1,3})^2} \quad \mathrm{r}_{3,4} := r_{1,2}$$

Derivation of electrical parameters

$$Lc_{1,1} := \frac{\mu l}{2\pi}\ln\left(\frac{r_{1,2}\times r_{1,3}}{r_{1,1}\times r_{1,4}}\right) \quad Lc_{1,2} := \frac{\mu l}{2\pi}\ln\left(\frac{r_{1,4}}{r_{1,3}}\right) \quad Lc_{2,2} := \frac{\mu l}{2\pi}\ln\left(\frac{r_{1,3}\times r_{3,4}}{r_{3,3}\times r_{1,4}}\right)$$

$$Cc_{1,1} := \frac{2\pi\varepsilon l}{\ln\left(\frac{r_{1,2}\times r_{1,3}}{r_{1,1}\times r_{1,4}}\right)} \quad Cc_{1,2} := \frac{2\pi\varepsilon l}{\ln\left(\frac{r_{1,4}}{r_{1,3}}\right)} \quad Cc_{2,2} := \frac{2\pi\varepsilon l}{\ln\left(\frac{r_{1,3}\times r_{3,4}}{r_{3,3}\times r_{1,4}}\right)}$$

$$Rc_{1,1} := \frac{\rho 1}{2\pi(r_{1,1})^2} \qquad Rc_{2,2} := \frac{\rho 1}{2\pi(r_{3,3})^2}$$

Star to delta transformation

$$Cd_{1,2} := \frac{Cc_{1,1}\times Cc_{1,2}}{Cc_{1,1}+Cc_{1,2}+Cc_{2,2}} \qquad Cd_{1,3} := \frac{Cc_{1,1}\times Cc_{2,2}}{Cc_{1,1}+Cc_{1,2}+Cc_{2,2}}$$

$$Cd_{2,3} := \frac{Cc_{1,1}\times Cc_{2,2}}{Cc_{1,1}+Cc_{1,2}+Cc_{2,2}}$$

Results

$$\frac{\mathrm{Lc}}{2} = \begin{pmatrix} 8.176\times10^{-7} & 1.609\times10^{-7} \\ 0 & 8.176\times10^{-7} \end{pmatrix} \text{ henry}$$

$$\mathrm{Cd} = \begin{pmatrix} 0 & 1.953\times10^{-11} & 3.844\times10^{-12} \\ 0 & 0 & 1.953\times10^{-11} \end{pmatrix} \text{ farad}$$

$$\frac{\mathrm{Rc}}{2} = \begin{pmatrix} 0.03 & 0 \\ 0 & 0.03 \end{pmatrix} \text{ ohm}$$