

# How far will it go?

**Roger Simms explains how you can determine the distance that a licence-exempt wireless telemetry link will cover.**

**W**ith the advent of deregulated low power data radio the economics of using traditional wire links for telemetry need to be examined. Licence-exempt data radio has low infrastructure costs, low installation cost and provides good system flexibility.

The uhf band between 410MHz to 480MHz has become internationally adopted for low power licence exempt use for digital data, telemetry and telecommand systems. It has the advantage of propagating in direct line of sight and will penetrate conventional build materials.

The rf signal fades quickly at the edge of its range. This factor allows multiple use of the same or adjacent frequencies in close proximity.

### International perspective

Although a common uhf band is used, national authorities have defined different specifications for licence exempt radio data transmissions.

They differ in the number of allocated rf channels, their bandwidth, spurious emissions and maximum rf power that can be transmitted. In the UK, the MPT1340 specification for operating on the 418MHz channel, allows one channel at 0.25mW rf power.

The radio range is limited to a few tens of metres. But the power consumption is low, as is the unit cost of the transmitters and receivers. Radios conforming to this specification are widely used in portable battery equipment or communicating with moving machinery.

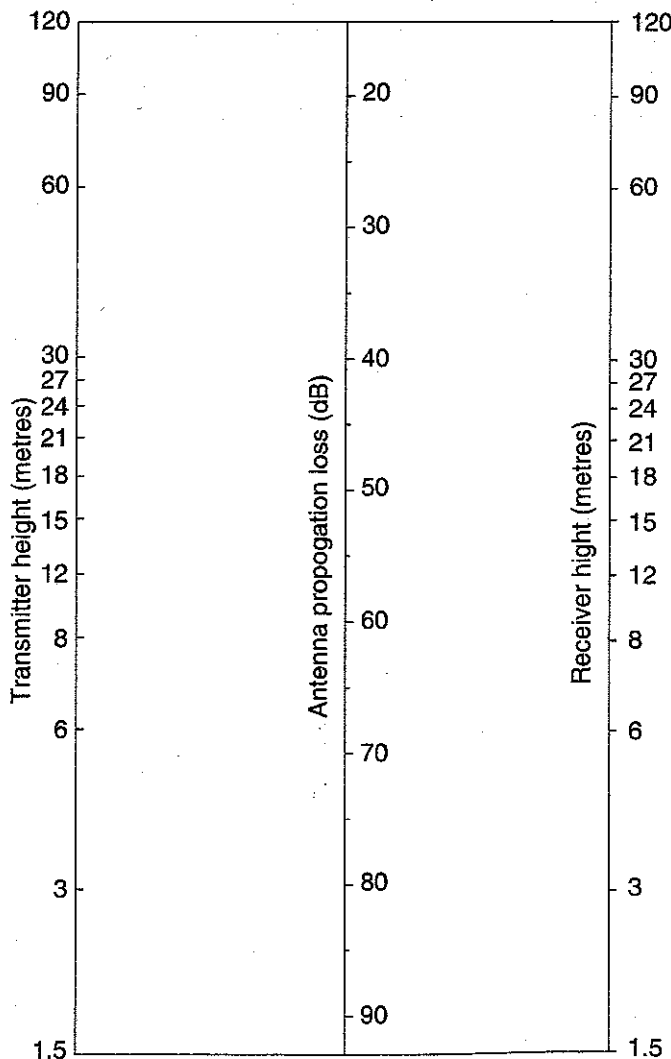
The UK MPT1329 specification covers operation at frequencies of 458.500MHz to 458.950MHz. Either 15 channels at 25kHz or 31 channels at 12.5kHz spacing are allowed with a maximum transmitter power of 500mW.

Radios using this band have data rates of up to 10kbit/s with good in building penetration and can achieve ranges of 10 to 20km, depending on the antenna configuration.

Most continental European countries adopt the ETSI 300-220 standard covering the 433.10MHz to 434.75MHz band with a transmitter power of 10mW. Data radios operating on this band have a range of up to 2km in free space and are used for short range data transfer.

The USA has various data and telemetry bands on frequencies

*Antenna propagation loss due to transmitter and receiver height. Line a rule up with the two antenna heights on the left and right-hand scales then read off loss at the rule crossing point on the middle scale.*



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between 440MHz to 470MHz with rf powers of between 100mW to 5W and are specified by FCC regulations CFR47 Pt2.

### Estimating radio range

Range is the most important parameter when assessing the practical implication of using a low power, licence exempt, radio system. It is sometimes difficult to correlate the transmitter rf power to the receiver sensitivity and estimate an effective range.

The main factors effecting the performance of a radio system are:

- Transmitter power
- Receiver sensitivity
- Terrain
- Antenna height
- Antenna feeder cable loss

UHF signals on the 410MHz to 470MHz band propagate directly between the transmitter and receiver and act in a similar way to light. There is therefore a maximum distance that a uhf signal can travel due to the curvature of the earth.

With both the transmitter and receiving antenna at a height of three meters and assuming there are no geographical obstacles, the radio horizon will be around 16 kilometres. If both antennas are raised to 100 metres the radio horizon would extend to around 90 kilometres.

With all licence-exempt radios, the rf power is strictly limited. The achievable range can therefore be much less than the radio horizon.

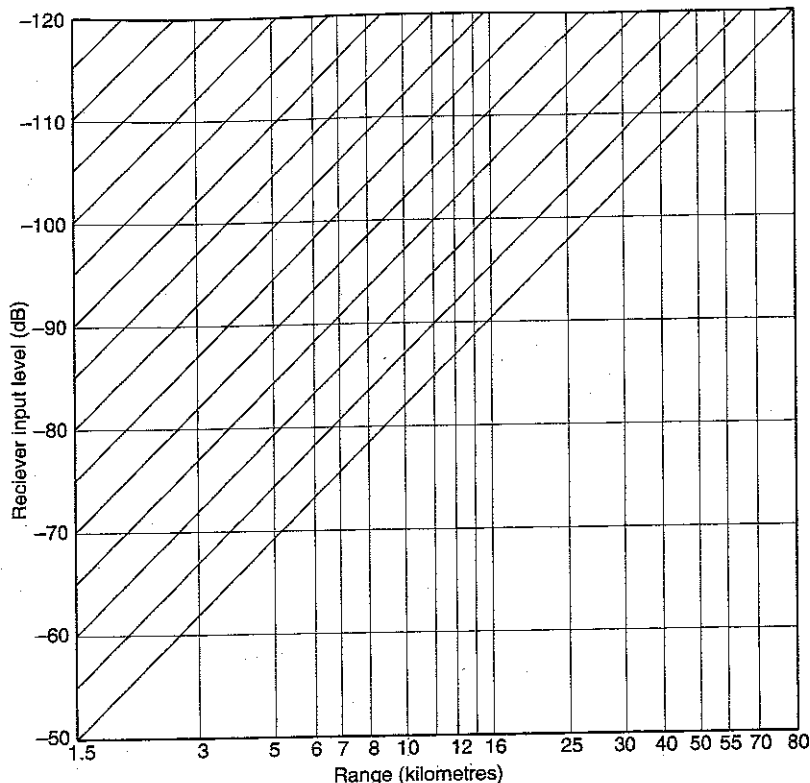
The radio range can be calculated by subtracting the factors causing the attenuation of the signal from the transmitter power. These include losses due to the antenna configuration, losses due to the terrain over which the signal will pass and the loss caused by the antenna feeder cable.

The propagation of the signal will depend on the height at which the receiver and transmitter antenna is above the ground. The higher the antenna the better the propagation.

Figure 1 correlates the height of both antennas to the expected propagation loss. The left and right scales give the height of the transmitter and receiver antenna. By placing a ruler between the two, a propagation loss can be estimated for any combination of heights.

Losses caused by the terrain can be estimated at around 50dB in open country or over water. This would be considerably more if the transmission path was to pass through buildings.

By subtracting all the propagation losses from the power irradiated from the transmitter, the required sensitivity at a near distance can be determined. The



*This chart is for finding the required receiver sensitivity at any distance from the transmitter.*

diagonal lines in Fig. 2 can then be used to determine the required receiver sensitivity at any given distance from the transmitter.

For example, if a licence exempt MPT1329 radio, radiating 500mW (27dBm) was transmitting at full power and both transmitting and receiving antenna were at 12 meters high, then from Fig. 1 the antenna propagation loss would be 55dB. If five meters of low-loss coaxial cable was used to connect the antenna to the transmitter and to the receiver there would be a further loss of 2dB.

Receiver sensitivity is the transmitter power minus propagation, terrain and antenna feeder losses. At approximately 1.5 kilometres would be -80dB assuming a 27dBm transmission, 55dB propagation loss, 50dB terrain loss and 2dB loss due to the antenna feeder.

Using the diagonal lines in Fig. 2 the required sensitivity of the receiver starting at -80dB can then be obtained at any distance from the transmitter. The maximum range using this configuration would be around 16 kilometres, given that the sensitivity of the receiver was as good as -120dBm.

### Installation criteria

Where ever possible, it is important to beam the rf signal by using a directional Yagi antenna. This reduces interference from other users that might be on the same channel. It also prevents the transmitter radiating its signal over more area than it needs to.

Yagi antennas have a specified power gain. Therefore the transmitting power must be adjusted to conform with both the licence exempt regulations and the power to which the transmitter has been type approved.

If care is not taken to make this adjustment then both the rf power and the spurious emissions will be amplified. This will cause rf pollution over the band, rendering nearby channels inoperable.

Before installing a licence exempt radio system it is also important to check that the intended channel is free. Most receivers have a relative signal strength indication (rssi). This gives a voltage output if the rf channel is in use. Hence a voltmeter can be used to check that no other signal is being transmitted on the frequency.

When a free channel has been found the receiver rssi signal can then be used to check the signal strength of the distant transmitter signal. This also provides a good method of finding the best position for the antenna and checking its alignment.

### In summary

Low power, licence-exempt data radio is a powerful alternative to wire over short and medium ranges. The cost compares well with dedicated telephone lines, data cabling in buildings and communicating with moving machinery. Added to this are the low cost of installation and physical flexibility afforded by radio communications. ■

# RMS

## watt, or not?

**When you see a power amplifier advertised as 100 watts rms, what - if anything - does it mean? Lawrence Woolf explains.**

**I** sometimes see the term 'watts rms' used in published text and advertisements. As 'rms' may be correctly applied to voltage and current it seemed worth while to examine the implications and meaning, if any, of applying it to power. This requires a degree of mathematics. Even if you are not interested in the maths, you might still find the summary interesting.

### DC power

If you apply a constant dc voltage to a fixed resistor, the current through the resistor and the power dissipated in it are easily calculated using Ohm's law,

$$I = \frac{E}{R} \quad (1)$$

where  $E$  is the applied voltage, or electromotive force,  $R$  is the resistance in ohms and  $I$  is the load current in amps. For power,

$$W = E \times I \quad (2)$$

or,

$$W = \frac{E^2}{R} \quad (3)$$

where  $W$  is the power dissipated in watts.

This power may be used in various ways but here we only need to consider that heat is generated. The rise in temperature that results from the power applied will depend on factors such as the power dissipated and the power radiated.

After a period of time a steady state is achieved. At this point, the radiated power balances the applied power and the load stays at a constant temperature somewhere above the ambient temperature.

As an example, a soldering iron takes some time to reach its working temperature and then should maintain it steadily.

### AC power

As far as heating the soldering iron is concerned, it does not matter whether the energy applied involves an alternating voltage or a direct voltage. The next task is to define the

alternating voltage that will supply the same heating energy as the direct voltage.

The problem is that the alternating voltage is, by definition, constantly varying. If the iron is powered by our mains at a frequency of 50Hz then each repeated sinusoidal cycle takes 20ms which is  $\frac{1}{50}$ th of a second, **Fig. 1a**). At time 0, the voltage is zero but rising.

After 5ms, the voltage reaches its positive peak, which I will call  $E_p$ . After a further 5ms, at 10ms, the voltage is back to zero but falling. At 15ms the voltage reaches its negative peak,  $-E_p$ . At 20ms the voltage is back to zero again and rising again as the sequence is repeated.

During this cycle the voltage has reached a positive peak and a negative peak. It has also been zero three times and has passed through every possible intermediate value twice. Which of these values, if any, could be used as a definitive value?

What is needed is a value that is numerically the same as for the direct voltage that will heat the iron to the same temperature. This is clearly not the peak value as, for most of the time, the magnitude of the voltage is below this.

We need to find a constant that we can multiply the peak value by to give the equivalent heating power of a known dc voltage. The constant seems likely to be less than one. This now raises the problem of also defining the alternating power which is also varying during the cycle.

In **Fig. 1b**) the ac voltage waveform is shown together with the power waveform. As the power is proportional to  $E^2$  both the positive and negative voltage peaks correspond to positive power peaks. When the voltage is zero, so is the power.

The resulting waveform is a raised cosine and has a frequency that is twice that of the voltage waveform. The load is assumed to be purely resistive so the power cannot, at any time, have a negative value.

A mathematical description is given by saying that,

$$E_t = E_p \sin(\omega t) \quad (4)$$

where  $E_t$  is the instantaneous voltage at time  $t$ ,  $E_p$  is the peak voltage and  $\omega$  is the angular frequency in radians per second, i.e.  $2\pi f$  where,  $f$  is the frequency in hertz.

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By using equations 3 and 4, with appropriate subscripts, the power waveform may be defined,

$$W_t = E_p^2 \times \frac{\sin^2(\omega t)}{R} \tag{5}$$

$$= E_p^2 \times \frac{1 - \cos(2\omega t)}{2R} \tag{6}$$

where  $W_t$  is the instantaneous power at time  $t$  and  $R$  is now the resistance of the soldering iron.

Equation 6 shows, by using a standard trigonometric substitution, that the power waveform is indeed a raised cosine at twice the frequency of the voltage waveform.

Figure 1c) shows that the average value of the power waveform is half the peak value. The waveform is symmetrical about the average power line,

$$W_{AV} = \frac{W_P}{2} \tag{7}$$

where  $W_{AV}$  is the average power and  $W_P$  is the peak power.

This average power is the heating power provided to the soldering iron and must be equivalent to the original dc power if it causes the iron to operate at the same temperature.

It is now possible to equate the dc and ac powers to derive the required constant to equate equivalent dc and ac voltages. Using equation 3, but with the peak ac values,

$$W_P = \frac{E_p^2}{R} \tag{8}$$

so that,

$$W_{AV} = \frac{E_p^2}{2R} \tag{9}$$

but,

$$W_{AV} = W_{dc} = \frac{E_{EQ}^2}{R} \tag{10}$$

where  $W_{dc}$  is the power from the dc source and  $E_{EQ}$  is the equivalent direct voltage.

We can now use equations 9 and 10 to find the relationship between the peak alternating voltage,  $E_p$ , and the equivalent direct voltage,  $E_{EQ}$ .

$$\frac{E_p^2}{2R} = \frac{E_{EQ}^2}{R} \tag{11}$$

This re-arranges to,

$$E_{EQ}^2 = \frac{E_p^2}{2} \quad \text{so that,} \quad E_{EQ} = \frac{E_p}{\sqrt{2}}$$

or

$$E_{EQ} \approx E_p \times 0.707$$

We now have our conversion factor that gives us the equivalent direct voltage that will produce the same heating effect as an alternating voltage, of known peak value, in a constant resistive load. This equivalent voltage is more commonly known as the rms voltage or  $E_{RMS}$  which now needs to be defined.

### Root-mean-square

I have now stated that when an alternating voltage is applied to a resistive load it will have the same heating effect, in that

load, as a direct voltage whose value is numerically the same as the rms value of the alternating voltage. I will next explain rms, and how to calculate it.

RMS is the abbreviation for root-mean-square. It is used in statistics as well as physics so is a useful concept. In order to apply it to a given waveform, such as a sine wave, rms can be considered in stages.

- Divide the waveform into narrow vertical slices, one is shown in Fig 1d). Each slice is narrow enough to consider it as having a single amplitude value.
- Square each value.
- Sum the values then divide the sum by the number of slices. You now have the mean of the squares.
- Finally, take the square root. This gives the square root of the mean of the squares of the sliced waveform.

The equation used in statistics is,

$$RMS \text{ value} = \sqrt{\frac{(x_1^2 + x_2^2 + x_3^2 \dots x_n^2)}{n}} \tag{12}$$

where  $x$  is the size of each slice or sample and  $n$  is the number of slices.

As we are considering a repetitive waveform that is easily defined mathematically there is a simpler way of performing the calculation. At least it is simpler for those of us who are familiar with integral calculus. The appropriate form of the equation is given by,

$$E_{RMS} = E_p \sqrt{\frac{1}{T} \int_0^T \sin^2(\omega t) dt} \tag{13}$$

where

$T$  is the time period under consideration.

The time period could be any that defines the symmetry of the waveform. For a sine wave just a quarter cycle is adequate as the following quarter cycles may be shown to be rotations or reflections of the first one. Therefore a suitable value for  $T$  is  $\pi/2$ , although the same result is achieved using  $\pi$  or  $2\pi$ .

In order to solve equation 13 we can put in this value and use the same trigonometric substitution used in equation 6.

$$\begin{aligned} E_{RMS} &= E_p \sqrt{\frac{2}{\pi} \int_0^{\frac{\pi}{2}} \frac{1}{2} (1 - \cos(2\omega t)) dt} \tag{14} \\ &= E_p \sqrt{\frac{2}{\pi} \left[ \frac{1}{2} \left( t - \frac{1}{2} \sin(2\omega t) \right) \right]_0^{\frac{\pi}{2}}} \\ &= E_p \sqrt{\frac{1}{\pi} \left[ \left( \frac{\pi}{2} - \frac{1}{2} \sin(\omega\pi) \right) - \left( 0 - \frac{1}{2} \sin(0) \right) \right]} \\ &= E_p \sqrt{\frac{1}{\pi} \times \frac{\pi}{2}} = \frac{E_p}{\sqrt{2}} \approx E_p \times 0.707 \end{aligned}$$

This is the same result as found in equation 11. Previously it was found by considering the symmetry of a sine wave. This result has been derived using the definition of rms and may be applied to any waveform.

A true-rms voltmeter displays the rms value even if the waveform is not a sine wave. However most ac voltmeters actually measure the peak value and are scaled to divide by  $\sqrt{2}$  even if the waveform is not a sine wave. Exactly the same argument applies to defining rms current as voltage.

**But rms power?**

Suppose we now apply the same calculation to the power function as we have done to the voltage function. We have found the ratio of  $E_p$  to  $E_{RMS}$  so we should be able to find the ratio of  $W_p$  to  $W_{RMS}$ .

Where we had to integrate a function involving  $\sin^2(\omega t)$ , we now have to integrate a function involving  $\sin^4(\omega t)$ . This is a little more complicated but there are standard trigonometric substitutions available that make the expression easier to handle.

Start by assuming that there is a meaningful relationship between  $W_{RMS}$  and  $W_p$ . From equations 5 and 8,  $W_p = E_p^2 \sin^2(\omega t) / R = W_p \sin^2(\omega t)$ . This leads to the assumption that,

$$W_{RMS} = W_p \sqrt{\frac{1}{T} \int_0^T \sin^4(\omega t) dt} \tag{15}$$

Again I am taking  $T$  as  $\pi/2$ . Using the same substitution as in equation 6,

$$\sin^2(\omega t) = \frac{1}{2}(1 - \cos(2\omega t))$$

therefore,

$$\sin^4(\omega t) = \frac{1}{4}(1 - 2\cos(2\omega t) + \cos^2(2\omega t))$$

also,

$$\cos^2(2\omega t) = \frac{1}{2}(1 + \cos(4\omega t))$$

so that,

$$\begin{aligned} \sin^4(\omega t) &= \frac{1}{4} \left( 1 - 2\cos(2\omega t) + \frac{1}{2}(1 + \cos(4\omega t)) \right) \\ &= \frac{3}{8} - \frac{1}{2}\cos(2\omega t) + \frac{1}{8}\cos(4\omega t) \end{aligned}$$

Substituting in equation 15 now gives,

$$\begin{aligned} W_{RMS} &= W_p \sqrt{\frac{2}{\pi} \int_0^{\pi/2} \left( \frac{3}{8} - \frac{1}{2}\cos(2\omega t) + \frac{1}{8}\cos(4\omega t) \right) dt} \\ &= W_p \sqrt{\frac{2}{\pi} \left[ \frac{3t}{8} - \frac{1}{4}\sin(2\omega t) + \frac{1}{32}\sin(4\omega t) \right]_0^{\pi/2}} \\ &= W_p \sqrt{\frac{2}{\pi} \left( \frac{3\pi}{16} \right)} = W_p \sqrt{\frac{3}{8}} \approx W_p \times 0.612 \end{aligned}$$

but from equation 7,  $W_{AV} = W_p/2$  or  $W_p = 2W_{AV}$ . This means that  $W_{RMS} = W_{AV} \times 1.225$ .

**In summary**

The implication of all this is that a transmitter that puts out 100W average power might also be said to have an output of 122.5W rms. This is hardly the same thing and would seem to have no practical or physical significance. It is merely a mathematical curiosity.

Alternatively one might assume that if someone claims an output of 100W rms then they are actually transmitting 81.63W average. The fact that it can be calculated does not,

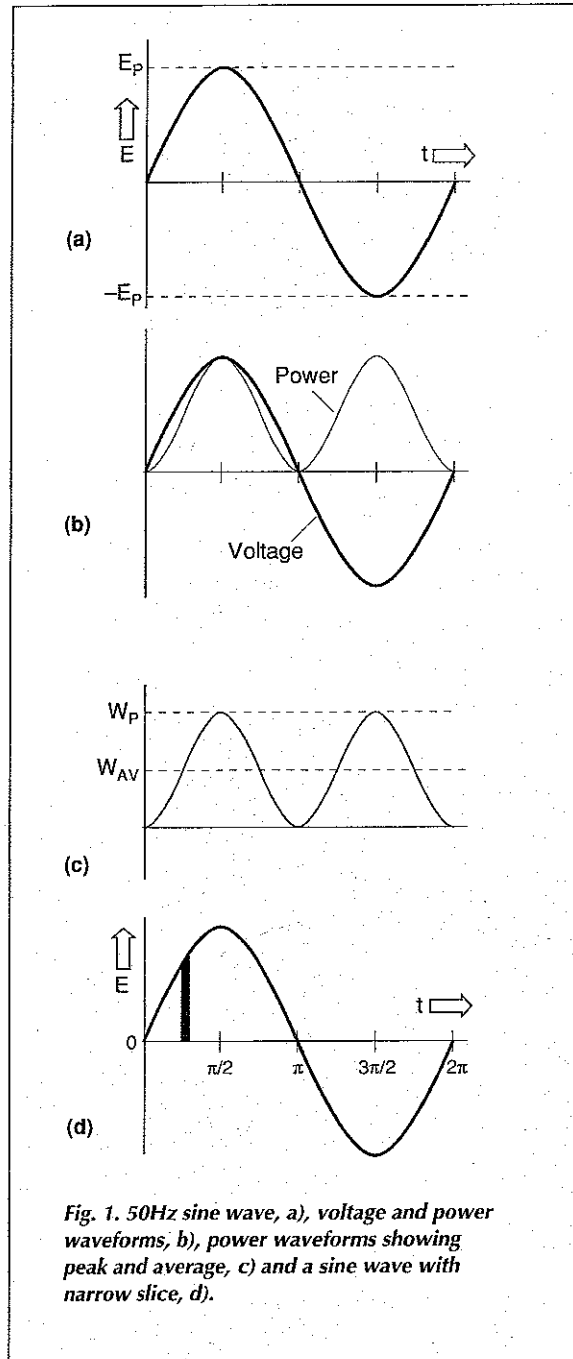


Fig. 1. 50Hz sine wave, a), voltage and power waveforms, b), power waveforms showing peak and average, c) and a sine wave with narrow slice, d).

in itself, imply that it could be useful.

The only useful result is that the product of rms voltage and rms current is average power. It is not rms power – even if it looks like a logical expectation. This is the same for mains frequency power, audio power and radio frequency power.

I suspect that those that use the term probably mean 'average power under continuous sine wave'. In this case a term such as 'continuous average power' would seem more appropriate, especially if an unregulated power supply is used.

If anyone knows of a genuine reason for using the term 'watts rms' then please let me know ([lawrence@itl.net](mailto:lawrence@itl.net)). ■