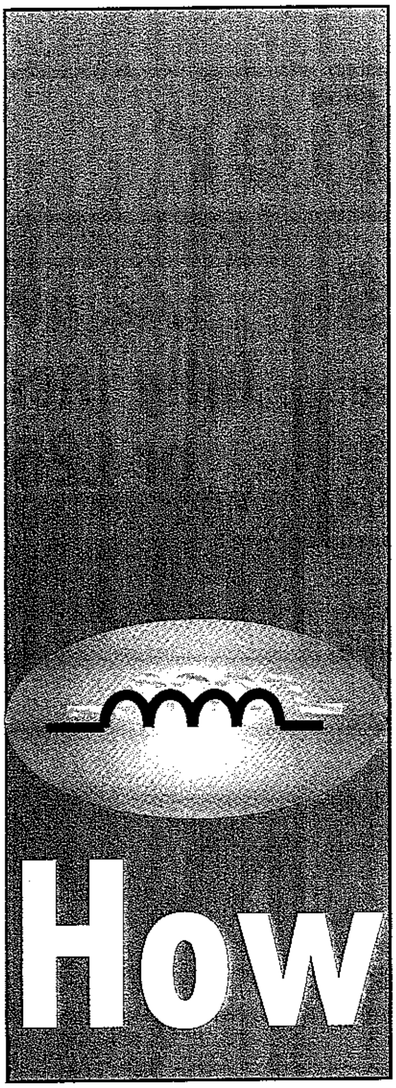

How long is *L*?

Inductance is the essential principle in applications as diverse as 50Hz power transformers, metal detectors and tuned circuits in mobile phones. But how many electronics engineers are truly conversant with it? Ian Hickman looks at inductance from the viewpoint of the practising rf engineer.

Over the years, the experienced design engineer develops a feel for the right value of a component in any circuit — at least to a first-order guess. In the case of inductors and capacitors, this guess is based on knowledge of the reactance of any particular component value at any given frequency.

My introduction to this practical aspect of electronic engineering was as a sandwich student in GEC's Central Research Labs at Wembley, which has long since disappeared. Someone was developing a rather sensitive circuit, and was troubled by 100Hz hum. Supply-line decoupling with a reasonably large electrolytic capacitor reduced it, but not enough.

A colleague jumped to his aid, brandishing a 1000μF electrolytic, exclaiming that if this looked like 160Ω at one cycle per second it should be pretty effective at 100 cycles. Hertz hadn't been invented yet.

The figure stuck in my then impressionable memory, and was subsequently embroidered upon. So: it follows that 1μF has a reactance of 160Ω or so at 1kHz, and a *CR* lowpass or high-pass combination of 1MΩ and 1μF has a time constant of 1s. Hence it will be 3dB down at 0.159Hz, while 1nF and 1kΩ will be 3dB down at about 160kHz.

And any budding rf engineer working with mobile phone technology would do well to remember that at 1GHz, a 1pF capacitor has a reactance of just 160Ω.

About inductance...

It is also handy to have a feel for the reactance of inductors of course – even though they are not so readily available in a wide range of close tolerance values, unlike resistors and capacitors.

Some years after graduation, I was working at a different firm, small but well known in the rf field. It produced, among other things, hybrids, baluns, attenuators and rf bridges. There, I soon developed a feel for inductance, similar to the feel I already had for capacitance. Just as the odd picofarad of strays can be disastrous at vhf and even more so at uhf, likewise too many nanohenrys in the wrong place can be a real headache.

To dimension the problem in practical terms, I experimented with an rf bridge, to get a feel for the inductance of an inch or so of wire.

Finding myself now, thirty years later, working on L Band equipment, the odd bit of inductance here or there is of real consequence. So, having access to a network analyser the likes of which could only grace my home laboratory in my dreams, I decided to check up on my earlier measurements. These have led to a convenient *aide-mémoire*, similar to the 'reactance of a capacitance at 1Hz' figure mentioned earlier.

Fixed inductance?

Capacitors come ready made. You can't add some more dielectric to increase the capacitance for example, even if you would want to.

Inductors are different. Frequently, you have to make your own inductor, either as a printed spiral on a pcb, or as a 'curly' – a coil which is either a self-supporting air-cored type, or wound on a former.

Using a former, the inductance can be increased by a ferromagnetic core or 'slug'. This increases the flux density within the coil. But the return path for the flux is still in air, so the resulting increase in inductance may only be in the region of 10% to 40% – even with a slug of high permeability material.

The important thing though is that the value of inductance is now readily adjustable, e.g. for tuning purposes, where the inductor forms part of a tuned circuit. It may also mean that the required inductance can be achieved with a fewer turns, reducing the coil's 'copper loss'. If the reduction of copper loss outweighs the 'iron loss' due to the slug, the Q of the coil may even be increased – an additional benefit over and above tunability.

Where a larger inductance is needed in the available space, and/or screening of the inductor is required, a coil former complete with a ferrite or dust iron sleeve or pot can be used, in conjunction with the slug. Now, most of the flux path is in a material with a relative permeability μ_r of anything in the range 2 to 100 or more, so the inductance per turn will be greatly increased: or is it per turn squared?

Most of the flux path is within the core, yes, but not all. The design of the former and core will be such that, even with the slug set for maximum inductance, there is still an appreciable air gap in the flux path.

The air gap stabilises the inductance, at the expense of reducing it from the fully closed path condition. For the permeability of ferromagnetic materials is likely to vary somewhat with selection, temperature and life, especially if the winding of the inductor is carrying any dc.

A_L and all that

The case with hybrids, baluns and untuned rf transformers of all sorts is different. If there is no dc in the windings, then a fully closed flux path can usefully be used, and is always preferred. Thus, the necessary magnetising inductance for the primary can be achieved with the minimum number of turns, keeping down the copper loss.

With a fully-closed flux path in high permeability material, virtually all the flux will be contained within the core, with very little 'leakage inductance'. Apart from the question of screening, leakage inductance, which is associated with flux in air rather than in the core, is of no consequence as long as the flux links both primary and secondary.

The term leakage inductance is therefore usually reserved for inductance due to flux that links one winding but not the other. This adds a reactive component in the primary circuit, in series with the transformed secondary load, reducing the effectiveness of the transformer, especially at higher frequencies.

The inductance of two 'close-coupled' coils of inductance L_1 and L_2 connected in series and wound in the same sense is L_1+L_2+2M. Here, the mutual inductance $M=\sqrt{(L_1L_2)}$ and the assumption is made that there is no leakage, all the flux in each coil linking with all the turns of the other.

The reason for the $2M$ is that M is the inductance due to the flux of coil L_1 which links L_2, to which must be added as much again due to the flux of coil L_2 which links L_1. So if the two 'coils' consists of two identical turns on the same core, then if the inductance of each is L nanohenries while that of the two together is $4L$ nanohenrys. This is because the coils are closely coupled, so in this instance, $L_1=L_2=M$.

If there are three turns, the inductance will be $3L$ plus $6M$ or $9L$ all told, since the flux of each turn links with both of the others. So extending the argument, the inductance of N turns on the said core will be N^2L nanohenrys.

I have illustrated this graphically in **Fig. 1** for an eight-turn winding. Each dot represents an inductance L – or equally well, M. Because of this result, you will find the inductance A_L of, for example, a two-hole balun core quoted as the inductance of a single turn, i.e. as so many nH/turns^2.

A challenge

Prove that $n^2=n+{}_nP_2$. This sounds like one of those arid academic exercises from Chapter N of an old maths textbook. But there is an application, right here. In Fig. 1, the left-hand column represents $8L$, due to the eight individual turns.

The seven dots in the left-hand column of the lower triangle represent the inductance due to the flux from turn 1, linking with turn 2, with turn 3, and so on to turn 8.

Likewise, the next column represents $6L$ – or strictly speaking $6M$, which is the same thing here. This is due to the flux in turn 2 linking with turns 3 to 8, and so on, totalling in all $28L$. But the total due to mutual inductance is twice this, since turn 2 links with turn 1, as well as turn 1 with turn 2.

This is represented by the upper inverted triangle. At a glance, you can see from the diagram that the grand total is $64L$.

It is also clear from Fig. 1 that $2(7+6+\ldots+1)=56$ – the block representing the two triangles – is equal to $7\times8=T$ where T is the total. Now $T=(8\times7\times6\times5\ldots\times1)/(6\times5\ldots\times1)$, which is conventionally denoted by the shorthand ${}_nP_2$, where P stands for permutation. Add the column on the left, and QED.

There's more on permutation in the separate panel.

How long is a piece of inductance?

So what has all this to do with the inductance of a length of wire? Well, it turns out that the inductance of a length of wire depends on just how you measure it. The simplest way is as a single air-cored turn.

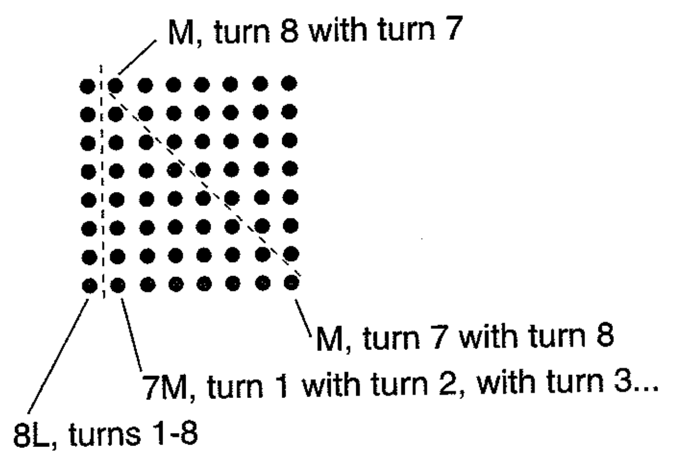

Fig. 1. Illustrating how the inductance of an eight-turn winding on a high permeability core is 64times that of a single turn.

The fact is, you can't measure the inductance of an isolated straight piece of wire. For the phenomenon of inductance to manifest itself, a current must flow. The current can only flow in a complete circuit, so even if you make a jig to hold a length of straight wire under test, what you actually measure is the inductance of the wire plus that of the jig.

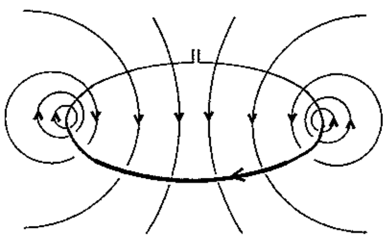

Fig. 2. Showing the lines of flux surrounding a current carrying wire.

Figure 2 shows a length of wire W centimetres carrying a current, and the resultant lines of magnetic flux. Some of these are only shown in part, as the ones passing through the turn near the centre are very long. The flux is naturally more concentrated near the wire, where the flux paths are shorter.

Perms and coms

Permutations and combinations are conveniently expressed by means of factorials. Factorial N, written $N!$, means $1\times2\times3...\times(N-1)\times N$. So factorials 4! is $1\times2\times3\times4=24$.

The permutation ${}_nP_r$ represents the number of ways of selecting r things from a group of n, the order being significant. So imagine for instance selecting four ornaments to range on the mantelpiece, from a choice of nine. For the item on the left there are nine possible choices. For the next, you have eight possibilities, seven for the third and six for the fourth.

The total number of ways therefore is 3024. This is equal to $n!/(n-r)!$, where here, n=9 and r=4.

The combination ${}_nC_r$ represents the number of ways of selecting r things from a group of n, the order not being important. So for instance the number of ways of selecting four apples to put in your shopping bag from nine on the greengrocer's shelf is smaller than ${}_9P_4$.

As there are 4! ways of arranging the 4 apples, ${}_nP_r$ is simply too large an answer, by a factor of 4! So ${}_nC_r={}_nP_r/r!=n!/\{(n-r)!\times r!\}$

I'm told that a perm on the football pools is not a perm, it's actually a combination, but never having done the pools, I wouldn't know.

Measuring a length of wire

What should the inductance be? In Fig. 2, if the length of the wire forming the loop were halved, the area of the loop through which all the flux must pass is reduced to a quarter of the previous figure. So it looks as though the reluctance S of the magnetic circuit – the equivalent of resistance R in an electric circuit – must be quadrupled.

Now inductance equals N^2/S, or here, with just a single turn, equals $1/S$. So the inductance of a length of wire, as a single-turn loop, would appear to be proportional the square of the length. This is a somewhat surprising statement.

On the other hand, if a single loop of inductance L nanohenrys were formed into two tight-coupled turns of half the circumference, S being quadrupled, each would then have an inductance of $L/4$.

By the earlier reasoning, the two together would have an inductance of $4(L/4)$. But would the inductance really be the same for one large or two small turns? Some careful mathematical analysis apart, there is only one way to find out.

Figure 3 shows an SMA connector terminating in an open spill, fitted with a solder tag, connected to the coil. The SMA plug was connected to a 'between-series-adaptor' which picked up on the APC7 connector on the network analyser's front panel, as indicated.

Puzzle corner

If you followed the argument about $n^2=n+{}_nP_2$ in the main article, you might like to try proving that $n^3=3n^2+{}_nP_3-2n$. If you need a hint, imagine layers like Fig. 1 stacked up on top of each other.

Is there a recurrence formula that will enable the value of n^{m+1} in terms of perms to be deduced from that of n^m?

Before connecting the coil, the test frequency of the *HP8753D* network analyser was set to 5MHz, and the measurement plane calibrated as the spill and solder tag used as the test terminals.

A 20cm length of wire was formed into a circle and connected to the terminals. The wire was 0.23mm diameter, measured over the usual self-fluxing insulation; this means it was probably 36SWG.

The wire was actually bifilar, with two parallel untwisted strands glued together side by side along their length. Each strand has a different colour enamel. This type of wire is available from the better stockists, and is very convenient when winding baluns, Ruthroff type line transformers and the like.

Various connections were tried, the results being recorded in **Table 1**. The final measurement was with the test terminals shorted.

Table 1. Frequency 5MHz. Bifilar wire. 36SWG. Length 10cm – plus small allowance for soldered connections.

Meas.	Conditions	Impedance (Ω)	Q	Q'	Inductance (nH)
1	Both strands in parallel	0.195+j 2.88	14.8	30	91.5
2	As 1, but one turn disconnected	0.265+j3.19	12.0	19	101.3
3	The two turns in series	0.455+j11.4	25.1	32	363.0
4	As 1, but connections reversed for one turn	0.185+j0.33	1.8	4	10.7
5	As 2, but disconnected turn removed	0.249+j3.26	13.1	22	103.6
6	2t in series as 1t of 2×dia, (20cm circ.)	0.385+j7.41	19.2	26	235.5
7	Terminals short circuited	0.100-j0.09	–	–	

The outcome

The short is close to perfect – at least as far as the reactive term goes. It actually looks like a very large capacitance, of such low impedance as to be negligible relative to the measured inductance of the loop.

However, it measured as having 100mΩ in series with the small reactive impedance – although in principle the earlier measurement-plane calibration supposedly factored both terms out.

Consequently, in addition to the value of Q calculated for the coil from the measured R and X figures, a corrected value is given in column Q′, with 0.1Ω subtracted from the measured value of *R*. Which figure is the more nearly correct is a moot point; it is probably somewhere in between.

Comparing row 5 with row 3 shows that two turns do not give four times the inductance of a single turn. This is because the shortest lines of flux surrounding each wire manage to sneak through the two thicknesses of insulation separating the turns.

This also happens when a high permeability core is present, as in a transformer. But the effect is then barely noticed, due to the much-enhanced flux linking both turns, resulting from the much greater flux density in the core. In fact, the leakage inductance is indicated by row 4, where the two turns are in parallel, but with the connections to one reversed.

With perfect coupling, the inductance would of course have been zero.

Was it the right figure?

The results agree reasonably well with my previous idea that an inch of wire looks like 25nH; those of you brought up metricated might find 1nH/mm an easier figure to bear in mind.

At 0.915nH/mm, the figure in the first row of Table 1 for two strands of 36SWG in parallel is about 10% lower. But the figures in the second and fifth rows are nearer. Comparing these results with row 3 shows that for an air-cored coil, lacking the flux concentration provided by a high permeability core, the inductance falls short of being proportional to N^2.

As a check, I repeated the experiment with 28SWG wire. Firstly, I measured a loop of 20cm of 28 gauge EnCu wire, result 201nH. This was then formed into two turns of exactly half the diameter, a couple of millimetres of extra wire at each end of the 20cm having been allowed for the soldered connections: result, 306nH.

One of the two turns was then removed, the measurement returning 86nH. Finally, as a sanity check, the terminals were measured shorted, this time the answer being 2.5nH. And for good measure, the inductance of 1 turn of 20cm circumference 16SWG wire was also determined. The results for the three gauges are compared in Table 2.

The results confirm the story told by Table 1. In particular, rows 5 and 6, like the figures 201nH and 86nH above, show that the inductance of a length of wire is certainly not proportional to the square of its length. Nor is it simply proportional to the length either, but somewhere in between.

In fact, the figures show that a single turn air-cored coil of twice the circumference will have about 2.33 times the inductance.

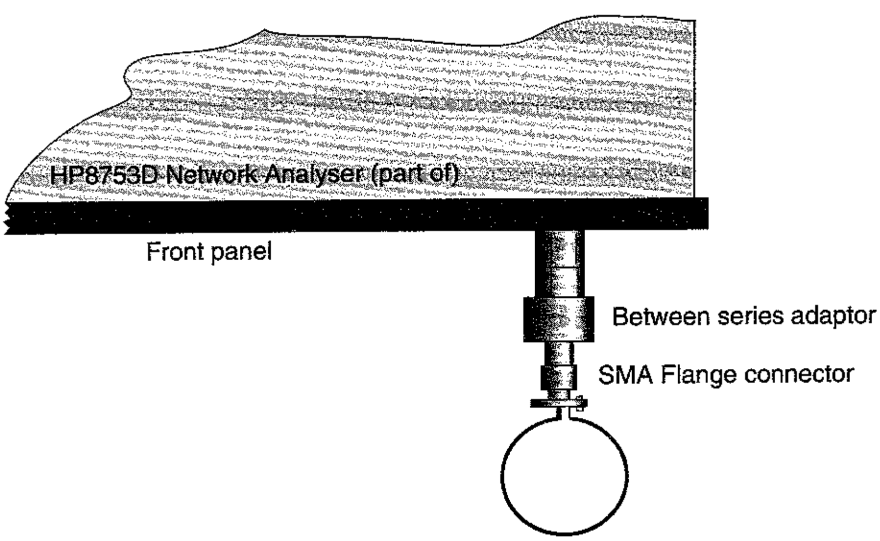

Fig. 3. The test set-up used to record the results in Tables 1 and 2.

The final question

The other question I set out to answer was what happens to the inductance if the wire of a single turn coil is refashioned into a two-turn coil of half the diameter. Why did the inductance go up from 201nH to 306nH when the single turn was fashioned into two smaller ones – or 235.5nH to 363nH in the case of Table 1?

Well, it is true that the reluctance *S* of that part of the magnetic path where it passes through the loop is quadrupled due to the smaller area, but that does not apply to the path as a whole. So the overall increase in reluctance is less than four times. The figures show that when a single turn is rewound as two tightly-coupled smaller turns, the inductance increases by 53%.

This is the result of two separate effects; the increase in reluctance of the magnetic circuit, and the leakage resulting in less than perfect coupling. The effect of the leakage is dramatically shown by the measured results for 1 and 2 turns of 86nH and 306nH (28SWG wire, above) or 91nH and 308nH, Table 1.

So, if you are talking about 28SWG or thereabouts, my earlier mental note that an inch of wire equals 25nH equals 16Ω at 100MHz was not far out. They are of course reactive or +jΩ, assuming the inch of wire has a reasonable Q, as it usually will.

Now 16Ω is only a rough estimate; it depends on the gauge of the wire, a thin wire having greater inductance per unit length than a thick one. In the mean time, I still reckon 25nH/inch is a good figure to bear in mind.

Although I do think that having a feel for the *reactance* per unit length at any given frequency is more useful in practical terms than the *inductance* per unit length. ■

Table 2. Frequency 5MHz, length 20cm, again plus small allowance for soldered connections.

SWG	36	28	16
Inductance, 1 turn	235.5nH	201nH	142nH